Unraveling and Inefficient Matching

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Abstract

Labor markets are said to unravel if the matches between workers and firms occur inefficiently early, based on limited information. I argue that a significant determinant of unraveling is the transparency of the secondary market, where firms can poach workers employed by other firms. I propose a model of interviewing and hiring that allows firms to hire on the secondary market *as well as* at the entry-level. Unraveling arises as a strategic decision by low-tier firms to prevent poaching. While early matching reduces the probability of hiring a high type worker, it prevents rivals from learning about the worker, making poaching difficult. As a result, unraveling can occur even in labor markets without a shortage of talent. When secondary markets are very transparent, unraveling disappears. However, the resulting matching is still inefficient due to the incentives of low-tier firms to communicate that they have *not* hired top-quality workers. Coordinating the timing of hiring does not mitigate the inefficiencies because firms continue to act strategically to prevent poaching.

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1 Introduction

Labor markets in which matches between firms and workers occur inefficiently early due to limited information are said to have unraveled. A classic example is the market for appellate court judicial clerks in the United States. Judges rush to make offers to law students as early as two years before the start date, at which point information about a student's legal writing is non-existent (Avery et al. [2001]). Unraveling is often ascribed to the combination of intense competition for scarce, high-quality workers and applicant uncertainty, which drives them to accept early offers (Roth and Xing [1994]).

However, there are markets that experience unraveling where employers *do not* face a scarcity of talent. In corporate law associate hiring, students also receive offers in their first year of law school. In investment banking, new analysts obtained their offers as college sophomores.[1] Table 1 lists notable labor markets and whether they unravel.[2]

Markets	Unraveling
Corporate Law Associates	✓
Private Equity Associates	✓
Investment Banking Analysts	✓
High-end Chefs and Line Cooks	✓
Hedge Fund Traders	X
Assistant Professors in Economics	X
Management Consulting	X
Programmers and Software Engineers	X

Table 1

[1]Judges fear a talent shortage because they seek those who have served on law review. At Harvard, cohort size is ≈ 580 students, but only 15% will ever serve. In corporate law and investment banking, many firms impose limits on the number of hires from each university (i.e., in 2014, the Merrill Lynch sales and trading division in New York capped Stanford hires at two). Given the size of an incoming analyst class is 40, it is difficult to believe that the bank is concerned about a scarcity of talent.

[2]See Ginsburg and Wolf (2004) for unraveling in law markets. SearchOne (recruiting firm) and the HR group at Merrill Lynch provided information on unraveling in investment banking. Herald Chen, former head of the TMT group at KKR, provided testimony on unraveling in private equity. Hiring timelines were provided by the Stanford and Columbia University career centers.

This paper identifies a novel channel by which unraveling can occur even in markets with an abundance of talent. I argue that unraveling can be caused by the presence and characteristics of a *secondary market*, whereby firms may poach workers currently employed by other firms. Poaching is a prevalent form of rematching in many industries: investment banks can recruit analysts from other banks, law firms can attract associates and partners from competitors, universities can hire professors laterally, and large venture capital firms can poach startups from smaller venture capital firms during series funding rounds. In models with a single stage of hiring and matching, the main driver of firm behavior is the desire to acquire top talent. With a secondary market, firms must also be concerned with their ability to *retain* the talent they hire. A secondary market might mitigate unraveling because it allows for rematching, but this depends on its transparency. In fact, moderately transparent secondary markets can *promote* unraveling as early hiring prevents rivals from learning about the worker. When secondary markets are highly transparent, unraveling disappears, but inefficiencies remain because of the threat of poaching.

These conclusions are based on a model of firm interviewing and hiring when there is a secondary market that allows firms to hire laterally instead of at the entry-level. I consider a situation with a high-tier and low-tier firm, each in need of a single worker from a large pool of applicants. Workers are of high or low type.[3] Time is divided into two stages. The first is the *primary market stage*, and the second is the *secondary market stage*. The former is analogous to the time students are in law school, while the latter is the time they work as associates post-graduation. In the primary market, firms can select a time to interview candidates and make a hiring decision; the worker begins working at the end of the primary market. The later a firm interviews, the better it can distinguish between them. A key feature of the model is that firms can choose whether to hire in the primary market or the secondary market, where they can monitor the worker hired by its competitor and "poach" her. This reflects the fact that firms receive signals about the quality of workers employed *at other firms*. The clarity of

[3] The model applies to other two-sided-matching markets. For example, "worker" could be replaced by "startup" and "firm" by "venture capitalist".

these signals varies across industries.[4]

The existence of a monitoring technology and an additional stage of mobility introduce a strategic element overlooked in prior work. Firms now must be mindful of losing a worker in the secondary market. To forestall poaching, a firm has two levers at its disposal: increase the hired worker's wage or increase uncertainty about the quality of the hired worker. Raising the wage makes the worker more costly to poach. Similarly, obscuring the quality of the worker it hires discourages poaching due to the increased uncertainty about the worker's ability. In my model, I demonstrate that when the secondary market is not too transparent, the best way to prevent poaching is via the latter action.

Consequently, unraveling arises because early hiring acts as a signal-jamming mechanism. Low-tier firms interview early to make monitoring and poaching in the secondary market more challenging for high-tier firms. As the secondary market becomes more transparent (i.e., the monitoring technology improves), unraveling disappears in equilibrium due to the low-tier firm's incentive to communicate that they have *not* hired top-quality applicants. Low-tier firms interview candidates at the end of the primary market to ensure the hiring of applicants that are unlikely to be of high quality. This has stark welfare implications. A highly transparent secondary market decreases total match quality as it creates an adverse signaling incentive for the low-tier firm.

Could one improve match quality via coordination of the hiring times in the primary market? Not necessarily. Unraveling is a strategic response to the threat of poaching, and coordination on hiring time does not fully mitigate the threat. Moreover, such coordination may *reduce* match quality in comparison to the decentralized setting. This indicates that to increase ex-ante match quality, the focus should be on the secondary market rather than controlling timing in the primary market. Finally, my analysis has consequences in other markets where assets of uncertain quality are mobile, and counterparties must make costly investments to ascertain their quality. "Unraveling" occurs in these environments in the form of under-investment in screening.

[4]In the market for economics professors, it is easy for universities to monitor professors at competing institutions: papers are published, and research is presented. On the other hand, it is more difficult for corporate law firms to ascertain the ability of associates at rival firms.

In the next section, I describe my model and its relation to the relevant literature. Sections 3 – 6 focus on the equilibrium analysis and Section 7 discusses applications of my results. Finally, Section 8 examines the robustness of the model with regards to adding additional firms. The qualitative features of my main results survive. Proofs are relegated to the appendix.

2 Model

2.1 Overview

Two firms, F_H (high-tier) and F_L (low-tier), each need a single worker from a finite pool of size N. Workers are either of high or low type, represented by $\theta \in \{H, L\}$. Each worker prefers to work for the high-tier firm, all else equal. The probability a worker is of high type is β, independent of the others. I assume N is sufficiently large so that if types were realized, there would be more high type workers than available slots with probability sufficiently close to 1. For expositional purposes I assume that the probability is exactly 1 (corresponding to the case where $N \to \infty$). [5]

Time is continuous from $[-T, \infty)$ and divided into two stages: $[-T, 0]$, which I call the primary market stage, and $(0, \infty)$, which I call the secondary market stage. Hiring can take place in each of the stages. If a firm approaches a worker at time $t \in [-T, 0]$ with an offer at wage w, and the worker accepts, the worker exits the market and *begins working at time 0*. A firm that fails to hire in the primary market can choose to 'poach' the employed worker at any time $t \in [0, \infty)$ in the secondary market.

2.2 Information in Primary and Secondary Market Stages

Primary Market

A firm choosing to hire in the primary market selects a time $t \in (-T, 0]$ at which to conduct interviews. Interviews are more informative the later they occur. One can think

[5] The probability that there are less than two high-type workers in the population converges to 0 as N gets large. All results in this paper hold for a sufficiently large finite N. See Appendix A.

of interviews as a sequence of progressively more informative binary tests that return a high or low-signal depending on the worker's true type. This can be represented by a function $M : [-T, 0] \longrightarrow [0,1] \times [0,1]$ that maps interview time to the probability the worker is of high type conditional on a high-signal, and the probability the worker is of high type conditional on a low-signal, respectively.

More generally, consider any mapping $M : [-T, 0] \longrightarrow [0,1] \times [0,1]$ satisfying:

1. $M(t) = (M_{high}(t), M_{low}(t))$.
2. $M_{high}(t)$ is increasing in t and $M_{low}(t)$ is decreasing in t.
3. For any $t > t'$, $M_{high}(t) - M_{high}(t') > 0$ if and only if $M_{low}(t) - M_{low}(t') < 0$.
4. $M_{high}(-T) = M_{low}(-T) = \beta$.

Such a mapping M is a reduced form representation of how well firms can sort workers at time t. The maximum probability that a worker is of high type given the results of any screening mechanism at time t is $M_{high}(t)$. The minimum probability that a worker is of high type given the results of any screening mechanism at time t is $M_{low}(t)$. Condition #4 indicates that there is no ability to sort at the beginning of the primary market. In Appendix C, I show that any such M is equivalent to a sequence of progressively more informative binary tests. Thus, I define a **high-signal** worker at time t to be a worker that is high type with probability $M_{high}(t)$. Similarly, a **low-signal** worker at time t is one that is high type with probability $M_{low}(t)$. At a given time t, firms can interview all workers in the primary market costlessly, which means they can hire a high-signal or low-signal worker almost surely.[6] Given sorting is best at the end of the primary market, $M_{high}(0)$ and $M_{low}(0)$ are the maximum and minimum probability with which a firm can be sure that it has hired a high type worker, respectively.

Fix an M. If a firm hires a worker at time $t < 0$ and the worker is a high-type with probability $p < M_{high}(0)$, I say that the market has **unraveled**.

[6]Under the binary test interpretation, randomization allows firms to hire a worker that is high type with probability $p \in [M_{low}(t), M_{high}(t)]$.

Secondary Market

Consider a firm that does not hire in the primary market, instead choosing to operate in the secondary market where it can monitor the worker hired by the other firm. The monitoring firm observes a signaling process yielding information about the employed worker's type. To formalize this, consider a worker whose probability of being of type $\theta = H$ is p_0. The monitoring technology is represented by an observable process $\{\pi_t\}$:[7]

$$d\pi_t = \mu_\theta dt + \sigma dB_t$$

$$\pi_0 = 0$$

One can interpret π_t as a noisy signal of visible worker output. The type-dependent drift satisfies $\mu_H \geq \mu_L$, reflecting expected differences in output between worker types. The quantity $\alpha = \dfrac{\mu_H - \mu_L}{2\sigma^2} = \dfrac{\bar{\mu}}{\sigma^2}$ represents the **transparency of the secondary market**.

2.3 Payoffs

Consider a type θ worker hired by F_i in the primary market at wage w. She will start working at time 0. Suppose at time t, firm F_{-i} approaches her with an offer of wage w'. If she accepts the offer, payoffs *from a time* 0 *perspective* are:

$$\left.\begin{aligned} \text{Worker}:\ & r\textstyle\int_0^t e^{-r\hat{t}}(w+\delta_{i=H})d\hat{t}\ + \\ & re^{-rt}\textstyle\int_0^\infty(w'+\delta_{-i=H})d\hat{t} \end{aligned}\right\}\ \delta \text{ is added payoff from working at } F_H.$$

$$\left.\begin{aligned} F_i:\ & r\textstyle\int_0^t e^{-r\hat{t}}(Z^i_\theta - w)d\hat{t} \\ F_{-i}:\ & re^{-rt}\textstyle\int_0^\infty e^{-r\hat{t}}(Z^{-i}_\theta - w')d\hat{t} \end{aligned}\right\}\ Z^i_\theta \text{ represents } \textit{match quality} \text{ to the firm.}$$

Match quality encapsulates productivity and output. I assume:

$$Z^H_H \geq Z^L_H \geq Z^L_L > 0 > Z^H_L$$

The inequalities reflect firm preferences and incorporate a notion of supermodularity in match quality. Both firms prefer high-type workers. Notably, the high-tier firm

[7] Construct a probability space $(\Omega, \mathscr{F}, \mathbb{P}_0)$, where $\mathbb{P}_0$ is the measure induced by p_0. Let B_t be a Brownian motion with respect to $\mathbb{P}_0$ independent of θ. The process $\{\pi_t\}$ is defined on $(\Omega, \mathscr{F}, \mathbb{P}_0)$.

never wants to employ a low type worker, while the low-tier firm finds such a worker acceptable. This is a natural assumption, as high-tier firms may have reputational concerns, so hiring a low-type worker is especially undesirable.

Workers not hired in the primary market receive a payoff normalized to 0 and leave the game. Firms that are unmatched receive a flow payoff of 0 for the duration they are unmatched. I assume that once a worker is hired and begins working for a firm, she can never be fired. This is without loss, as F_L will never choose to fire a worker, and giving F_H the power to terminate a worker is analogous to a rescaling of the match quality.

Lastly, I impose the trivial condition that $M_{high}(0)Z_H^H + (1 - M_{high}(0))Z_L^H > 0$, as otherwise the high-tier firm would never want to hire in the primary market.

2.4 Strategies

Due to continuous time, formal definitions of strategies require care. The technicalities, which I omit here, can be found in Appendix A. Each worker strategy consists of the following decisions: accept or reject offers in the primary market, and, if hired, whether to accept a lateral offer. F_L**'s strategy** consists of:

1. A time t in the primary market at which to interview.

2. Conditional on t, whether to make an offer to a high or low-signal worker.

The low-tier firm will always choose to hire conditional on interviewing (as it will be unable to "poach" a worker from F_H). It is clear F_H never enters the primary market before $t = 0$. Therefore, its decision is whether to operate in the secondary market or hire in the primary market at $t = 0$, and it can make this choice based on the observed time at which F_L hires and the offered wage. Upon this observation, F_H formulates beliefs about whether F_L hired a high-signal or low-signal worker. Thus, F_H**'s strategy** *maps the observed time at which* F_L *hires* to:

1. A probability of hiring in the primary market.

2. A poaching rule conditional on operating in the secondary market.

3. A belief about the worker hired by F_L.

Within the secondary market, the high-tier firm must decide at each time whether to hire the worker at F_L or not. Hence, such a decision is equivalent to a stopping time where F_H hires the worker when the process stops. Formally, a **poaching rule** is a stopping time τ adapted to the filtration $\mathscr{F}_t^\pi$.

A pair of firm strategies constitute an **equilibrium** if each firm is best-responding at each information set, and F_H's beliefs are consistent on-path.[8]

Lemma 2.1 *A worker that receives an offer in the primary market will always accept.*

Proof: *See Appendix A.*

One might think that a worker receiving an offer from F_L is a public signal of her type, allowing for the opportunity to strategically reject the offer. As N is large, there is no incentive to do so, as the probability of receiving a future offer is essentially 0.

2.5 Relation to the Literature

An extensive literature on market unraveling was spawned by Roth and Xing (1994), who identified the phenomenon and several dozen markets that had experienced an unraveling of appointment dates. Along with Avery et al. (2001), they conjecture that firms "jump the gun" to acquire top talent. Niederle et al. (2013) formalize this intuition in a market with *comparable* supply and demand, where firms and workers both believe that they are on the long-side of the market. My paper demonstrates that unraveling can occur *even when talent is plentiful*, and there is a supply and demand *imbalance*.[9]

A common theme in papers on unraveling is that there is informational uncertainty amongst the participants. Li and Rosen (1998) and Li and Suen (2000) examine matching markets with one-sided and two-sided uncertainty, respectively. In both cases, the

[8] The equilibrium concept used is Perfect Bayesian Equilibrium. As will be discussed in Appendix A.1, any Perfect Bayesian Equilibrium is also a Sequential Equilibrium.

[9] Other papers propose different causes. Damiano et al. (2005) examine a search and matching model, where introducing participation costs decrease the fraction of low-types searching in early periods. Firms are incentivized to match early or face a pool of workers bereft of talent. Halaburda (2010) and Echenique and Pereyra (2016) view unraveling similar to a bank run: unraveling by one firm incentivizes unraveling by others. Fainmesser (2013) highlights the effect of networks and social connections on unraveling.

authors find that unraveling acts as a form of insurance against remaining unmatched. In my paper, unraveling acts as a form of insurance against competitors poaching a hired worker. However, due to the threat of poaching, coordinating when contracting occurs does not necessarily increase match quality. Complementing these papers is Du and Livne (2016), which studies a model where agents can choose to contract early or wait and compete with an influx of new agents in the second period. They find that when transfers are flexible, unraveling is mitigated. My model allows for flexible wage-setting, yet unraveling is unabated due to the secondary market. Adjusting the wage does not provide the same strategic benefit that early matching yields.

The relationship between strategic signaling incentives and unraveling builds on Ostrovsky and Schwarz (2010), which endogenizes information revelation in the primary market to show that unraveling can be prevented through optimal information disclosure. However, they do not consider the presence of a secondary market stage, which *can counteract* the benefits of information disclosure in the primary market.

None of the above papers allow for rematching between workers and firms, which is a significant component of this paper. By dividing time into two stages, I can demonstrate the strategic signaling incentives induced by the presence of a secondary market. Moreover, my model yields a characterization of the exact time at which unraveling occurs and the development of precise comparative statics regarding hiring times.

My paper fits into a broader literature regarding the strategic incentives generated by aftermarkets and resale markets.[10] One can think of the secondary market in my paper as an aftermarket where extra information becomes available. A critical difference between my model and this literature is the common-value component and the fact that objects can become unavailable. From a labor market perspective, Milgrom and Oster (1987) develop a model where firms profit by placing talented workers in less visible positions to prevent wage increases from competition. They do not focus on unraveling or screening. In my model, a wage increase is beneficial in that it makes poaching more costly. Moreover, firms can not control employee visibility. Instead, firms can control the flow of information by affecting the initial signal of a worker's ability.

[10] Ausubel and Cramton (1999), Halafir and Krishna (2009), Carroll and Segal (2019).

2.6 Discussion of Assumptions

The first assumption is that primary market hiring prevents the competing firm from learning anything about the hired worker before the secondary market begins. This is the reality in many two-sided matching markets. For example, in hiring at the university level, once an offer is accepted, students are not permitted to interview with other employers.[11] The second major assumption is that N is "sufficiently large".[12] When N is large, I need not consider the case of all candidates failing or passing a given test at any stage in the primary market. The probability of such an event rapidly tends to 0. In addition, suppose F_L interviews candidates at time t, making an offer according to a known hiring rule. If F_H interviews the remaining applicants at a later date, its beliefs about the applicant hired by F_L will not be affected. This isolates the effect of the informativeness of the secondary market on firm behavior in the primary market.

3 Benchmark: No Secondary Market

Before discussing the dynamics of the model, I highlight the benchmark, which serves as a comparison to the equilibrium findings. Suppose no secondary market exists.

Proposition 3.1 *If a secondary market does not exist, there is no unraveling.*

Since N is large, the law of large numbers implies that there will be βN high types with probability close to 1. Therefore, if the firms interview at $t = 0$, each will be able to match with a high-signal worker almost surely. The total match value generated is:

$$M_{high}(0) \cdot (Z_H^L + Z_H^H) + (1 - M_{high}(0)) \cdot (Z_L^L + Z_H^L)$$

When there is no secondary market and an abundance of talent, there is no unraveling. This aligns with Roth and Xing (1994) and Niederle et al. (2013). Each firm can hire a high-signal worker at the end of the primary market.

[11] Reneging on an offer has significant repercussions.

[12] I discuss the precise threshold in the proof of Lemma 2.1.

4 Poaching

I begin by characterizing the optimal poaching rule *conditional on F_H operating on the secondary market*. That is, suppose F_L has hired a worker in the primary market, and F_H is monitoring the worker. If the worker is earning a wage w, to successfully poach at any time, F_H must offer $\max\{w-\delta, 0\}$. Thus, F_H's decision problem is:

$$\Gamma_H(p_0, w) = \max_{\tau} \mathbb{E}[e^{-r\tau}(Z_\theta^H - \max\{w-\delta, 0\})|\mathscr{F}_t^\pi, p_0, w]$$

To determine the optimal stopping rule for F_H, I map π_t to the space of posterior beliefs.[13] Given initial belief p_0, let $p_t = \mathbb{P}(\theta = H|\mathscr{F}_t^\pi)$ denote the posterior belief that the worker is of high type at time t given the observations from the process $\{\pi_t\}$.

Proposition 4.1 *The optimal poaching rule is a threshold stopping time of the form:*

$$\tau^* = \inf\{t \geq 0 : p_t \geq B^*\}$$

Where B^* depends on (α, w, Z_θ^H), is time-invariant, and independent of p_0.

Proof: *See Appendix B.*

The decision to poach depends solely on whether the belief about the worker is above a static threshold B^* that is independent of the initial belief p_0. The sharp characterization of τ^* elucidates the close relationship between poaching and the secondary market's informativeness. As the transparency of the secondary market increases (α increases), B^* increases. With a more informative signal, the high-tier firm can afford to wait for a higher posterior. Note that while w and α both affect the value of B^*, only α affects the *speed* of reaching a given threshold.

5 High-tier Firm Hires Laterally Only

To understand the incentives at work, consider a setting where the low-tier firm F_L is the only participant in the primary market, with the high-tier firm F_H only hires

[13]Related is the experimentation and hypothesis-testing literature (e.g. Wald [1947], Moscarini and Smith [2001]).

laterally. Out of the pool of available workers, F_L hires one and understands that she may eventually be poached by F_H.

The high-tier firm uses the poaching rule τ^*. Suppose the high-tier firm has initial belief $\tilde{p}_0$ about the worker F_L has hired. The payoff to the low-tier firm from employing a worker at wage w with probability p_0 of being a high type is:

$$\Sigma_L(p_0, \tilde{p}_0, w) = \underbrace{p_0(Z_H^L - w) + (1 - p_0)(Z_L^L - w)}_{\text{Expected Match Value}} - \textit{Loss due to Poaching}$$

The first term reflects the expected net match quality conditional on the low-tier firm keeping the worker forever. The second term is the expected loss due to poaching by F_H. The loss due to poaching is dependent on the actual probability (p_0) that the worker is of high type as well as the high-tier firm's belief ($\tilde{p}_0$) that the worker is of high type.

Of particular interest is the function $\Gamma_L(p_0, w) = \Sigma_L(p_0, p_0, w)$, which is the expected payoff to the low-tier firm when its belief is consistent with the high-tier firm's belief about the worker ($\tilde{p}_0 = p_0$). Considering Γ_L, observe that the two parameters that F_L can control are the prior on the worker it hires and the wage. Increasing the wage increases the cost of poaching for F_H, thereby raising the belief threshold B^* needed before poaching can occur. On the other hand, changing the probability that the worker hired is of high type delays the time until poaching. It is not obvious which lever the low-tier firm should pull.

Proposition 5.1 ***"Obscuring the quality of the worker is best."***

Fix an α. There exists p^* depending on α such that:

$$(p^*, 0) = arg \max_{p_0 \in [0,1], w \geq 0} \Gamma_L(p_0, w)$$

$$and \quad \frac{\partial \Gamma_L(p_0, 0)}{\partial p_0}\bigg|_{p_0 = p_0^*} = 0$$

Proof: *See Appendix B.*

Increasing the wage increases F_L's costs but also makes poaching more costly. By increasing w, F_L can artificially increase B^* and lengthen the expected time it employs a worker. However, what Proposition 5.1 shows is that the best way to make poaching more costly is to hire a worker with a different expected match quality and pay her a wage of 0. The key is to show that the wage is a suboptimal tool to deter poaching. Formally, Lemma B.4 in the appendix proves that:

For any p_0 and $w \geq 0$, there exists p_0' such that:

$$\Gamma_L(p_0', 0) \geq \Gamma_L(p_0, w)$$

The quantity p^* defined in Proposition 5.1 represents the optimal induced prior in a game where the high-tier firm is committed to hiring in the secondary market *and knows* the probability that the worker is of high type at the time it was hired by the low-tier firm. The intuition behind the lemma is that the benefit from increasing w occurs on the "back end". Given an initial threshold B^*, the gains from moving the threshold to $B^{*'} > B^*$ will only be seen when the high-tier firm's belief is in $[B^*, B^{*'}]$. On the other hand, reducing the initial probability with which the worker is of high type affects *the path to* B^*. Since future payoffs are discounted more heavily, elongating the "front-end" is more valuable. Thus, the optimal wage is 0, and F_L's optimal decision only involves the induced belief of the worker it hires rather than any wage consideration.

The proof of Proposition 5.1 also illustrates how the optimal induced prior p^* varies naturally with α, the transparency of the secondary market. As $\alpha \to 0$ (low transparency), the low-tier firm understands that poaching is more challenging; it is more willing to hire potentially high-quality workers in the primary market. Conversely, as $\alpha \to \infty$ (high transparency), F_L seeks to hire a high type worker with low probability to ensure that it can keep the worker for a long time.

Proposition 5.2 *There exists* $\bar{\alpha} > 0$ *such that:*

1. *For* $\alpha \in [0, \bar{\alpha}]$*, the market unravels. The low-tier firm hires at* $t^* < 0$*, where* $M_{high}(t^*) = p^*(\alpha)$*. The wage is zero.*

2. *For* $\alpha \in (\bar{\alpha}, \infty)$*, any equilibrium involves the high-tier firm believing the hired worker is high type with probability* $p(\alpha)$*, where* $p(\alpha)$ *is the belief that makes the low-tier firm indifferent between worker types. The low-tier firm mixes between hiring high and low signal workers to induce the probability* $p(\alpha)$ *that the worker is high type. Both unraveling and non-unraveling equilibria exist.*

Proof: *See Appendix B.*

6 Equilibrium and Match Quality

6.1 Equilibrium

The previous section examined the strategic decisions made when F_H specializes in hiring in the secondary market. I now analyze the equilibrium dynamics when F_H can choose whether to hire on the primary market *as a function of the history it observes*.

The decision to operate on the secondary market depends on the effectiveness of screening in the primary market. Suppose at the end of the primary market that the screening ability is such that the posterior belief is already above the poaching threshold. In that case, the high-tier firm will not hire in the secondary market. Since the labor market supply is large, both firms will interview at $t = 0$ and hire a high-signal worker.

OBSERVATION 6.1 *If $M_{high}(0) > B^*$, the market does not unravel.*

Proof: *See Appendix B.*

In observation 6.1, if the threshold belief for poaching, B^*, is lower than the belief about a high-signal worker at the end of the primary market, the high-tier firm will always choose to hire in the primary market. In this case, the secondary market provides no value to the high-tier firm; the monitoring technology does no better than what can be achieved with screening in the primary market. This serves as the basis for a definition of opaqueness. Let α_{opaque} denote the value of transparency (α) such that $B^* = M_{high}(0)$. Hence for all $\alpha \leq \alpha_{opaque}$, the high-tier firm will never operate in the secondary market.

At transparency levels $\alpha > \alpha_{opaque}$, the insight that the secondary market incentivizes the low-tier firm to hire in a manner to prevent poaching still holds. However, recognize that the high-tier firm will not necessarily ex-ante commit to operating in the secondary market. Since the high-tier firm can only observe the time at which the low-tier firm hires, its belief about the worker hired can only be contingent on the hiring time t and its knowledge of the interviewing technology $M(t)$.

It becomes crucial to pin-down the firms' "indifference beliefs":

1. When is the high-tier firm indifferent between operating in the secondary market and hiring at the end of the primary?

2. Conditional on the high-tier firm operating in the secondary market, what belief does it need to have to make the low-tier firm indifferent between worker types?

Let $\bar{p}$ be the value such that F_H is indifferent between primary market hiring and hiring on the secondary market when F_L hires a worker of type $\bar{p}$. Likewise, p_{ind} be the belief such that if F_H was hiring on the secondary market and believed F_L's worker was high type with probability p_{ind}, F_L would be indifferent across worker types.

The particular equilibrium and its characteristics will depend on the values of $\bar{p}$ in relation to p_{ind}. The qualitative results are provided below in Theorem 6.2. In Appendix B, I list all the equilibria that exist as a function of these quantities. Importantly, for a large class of the parameter values, the set of equilibria will reduce substantially.

For intuition, I describe an equilibrium that will emerge: a **pure strategy unraveling equilibrium**. Consider the situation where the high-tier firm *would not* want to operate in the secondary market if F_L hired a worker that was high type with probability p^*. Hence, $\bar{p} > p^*$. Therefore, in any pure strategy equilibrium, it must be the case that the low-tier firm hires at $\bar{t}$ to induce belief $\bar{p} \geq p^*$ such that the high-tier firm is indifferent between operating in the secondary market and hiring in the primary market. When $\bar{p} \in (\beta, M_{high}(0))$, a **pure-strategy unraveling equilibrium** exists. The low-tier firm interviews at $\bar{t} < 0$ where $M_{high}(\bar{t}) = \bar{p}$. The high-tier firm's equilibrium strategy is to hire on the primary market if the low-tier firm hires at $t \leq \bar{t}$ and to specialize in secondary market hiring if the low-tier firm does not hire by $\bar{t}$.

This equilibrium outcome is salient because it arises in many of the unraveled industries described in the introduction: the market for corporate law associates, private equity associates, and investment banking analysts. In this equilibrium, the wage is generally close to 0. Since the low-tier firm hires a high-signal worker at time $\bar{t}$, the only way to do better is to hire at a later time and pay a higher wage to deter poaching. By hiring at a different time, though, the high-tier firm's belief will also adjust, reducing any gains in such a deviation. That the equilibrium wage is close to 0, while surprising, is consistent with the observation that there is little variation in entry-level salaries in many of the unraveled industries discussed. For instance, in corporate law, private equity, and investment banking, entry-level salaries are fairly uniform across firms.[14][15]

The next theorem summarizes the qualitative features of the various equilibria. Since the "indifference beliefs" are determined by the transparency level α, I can map the values of α to the types of equilibria that arise.

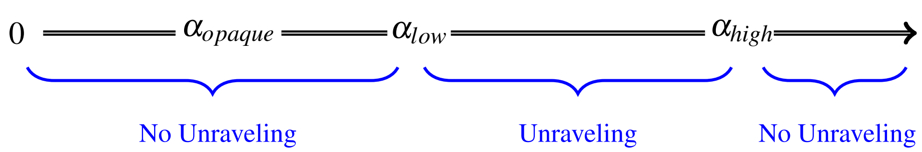
Theorem 6.2 *There exist thresholds $0 \leq \alpha_{opaque} \leq \alpha_{low} \leq \alpha_{high}$ such that:*

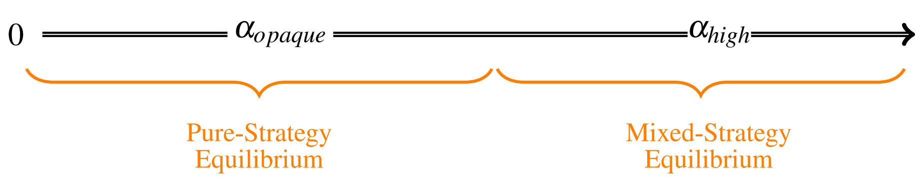
1. *For $\alpha \in [0, \alpha_{opaque})$, there is no unraveling in equilibrium and the wage is 0.*

2. *For $\alpha \in [\alpha_{opaque}, \alpha_{low}]$, there is no unraveling in equilibrium but the low-tier firm pays a positive wage $w > 0$.*

3. *For $\alpha \in (\alpha_{low}, \alpha_{high}]$, there is unraveling in equilibrium.*

4. *For $\alpha \in (\alpha_{high}, \infty)$, there is no unraveling in equilibrium.*





Proof: *See Appendix B.*

[14] National Association for Law Placement and North American Private Equity Compensation Survey.

[15] https://www.wsj.com/articles/starting-law-firm-associate-salaries-hit-190-000-1528813210

When the high-tier firm is exclusively hiring laterally, changing the wage is a sub-optimal lever to deter poaching (see Proposition 5.1). When the high-tier firm has a choice between hiring at the end of the primary market or operating in the secondary market, the low-tier firm can now use the wage to deter poaching. By increasing the wage, the low-tier firm can force the high-tier firm to hire at the end of the primary market. The low-tier firm's worker will never be poached and payoffs increase discontinuously. Therefore, wage flexibility can allow for some reduction in unraveling in the low transparency environments [$\alpha \in (\alpha_{opaque}, \alpha_{low})$]. However, it is not enough to entirely stop unraveling. Obviously, wages are bounded above by Z_H^L. Moreover, a wage w only serves as a potential deterrent when $w \geq \delta$, the intensity of worker preference for the high-tier firm. When δ is negligible, as transparency increases, even a wage of Z_H^L is not sufficient to mitigate poaching.

As the monitoring technology in the secondary market improves, the low-tier firm wants to induce a lower belief about the worker it hires to prevent poaching. Hiring a low type worker with high probability requires being able to sort very well. As a result, it chooses to hire at the end of the primary market. However, if the low-tier firm is screening to hire a low type worker with high probability, there is no incentive for the high-tier firm to operate in the secondary market. On the other hand, if the high-tier firm chooses to hire at the end of the primary market, the threat of poaching vanishes, and so the low-tier firm no longer has an incentive to hire the worker with a low-signal! Thus, to support the non-unraveling equilibrium when the secondary market is sufficiently informative, the low-tier firm hires at the end of the primary market *but mixes* between hiring a high-signal and low-signal worker. The high-tier firm mixes between operating in the secondary market and hiring at the end of the primary market.[16]

Recall that α_{opaque} and α_{high} depend on $M_{high}(0)$ and $M_{low}(0)$. In other words, the transparency thresholds are determined *relative* to firms' sorting ability in the primary market. In the extreme, if firms were able to perfectly distinguish between types in the primary market, then there would never be unraveling and $\alpha_{opaque} = \infty$. The trans-

[16]Since the optimal poaching rule is independent of initial beliefs, F_H's decision reduces to whether to operate on the secondary market or the primary market. It follows from Hendon et al. (1996) that any Perfect Bayesian Equilibrium will also be a sequential equilibrium.

parency thresholds are increasing in $(M_{high}(0), 1-M_{low}(0))$.

Furthermore, the value in sorting is not just about identifying high-types but the ability to identify low types as well. If β is very small so that $\bar{p} > \beta$ for all α, then as transparency increases, unraveling will not dissipate. Why? Because while talent is not scarce from a realization perspective, it is rare. Sorting to find a low-type worker with high probability is extremely easy: random selection yields a low type with probability $1-\beta$, which is high. Consequently, there is little incentive to sort to hire a low-quality worker. For β sufficiently small, $\alpha_{high} \to \infty$.[17]

One may wonder whether the equilibrium strategies are realistic depictions of firm behavior. That is, do top firms condition their decisions on whether a lower-tier competitor hired a first-year law student in February? In some matching markets, where matching processes are very public (e.g., Venture Capital funding), such strategies are indeed realistic. However, in labor markets, one should view the equilibrium strategies and outcomes as limit points of a long-run process that involves learning. Over time, a firm can observe the quality of workers at its competitors and deduce how well its competitors are screening.[18]

6.2 Match Quality

Using the characterization of the equilibria in Theorem 6.2, I compare the total equilibrium match quality to the benchmark-setting where there is no secondary market.

In Theorem 6.3, I provide weak conditions that are sufficient for the total match quality in any equilibrium to be lower than the benchmark-setting. Under weak conditions on the relation between the payoffs and maximal sorting in the primary market, I can exclude the other types of equilibria described in Appendix B. These conditions rule out certain “corner cases".

[17]The threshold values are necessarily distinct when $\beta Z_H^H \geq p_{max} Z_H^H + (1-p_{max}) Z_L^H$.

[18]This interpretation echoes Green and Porter (1984), where firms can not observe competitors' prices, but instead see noisy estimates of demand, which they use to deduce said prices.

Theorem 6.3 *Suppose the following conditions hold:*

1. $M_{low}(0)Z_H^H < M_{high}(0)Z_H^H + (1 - M_{high})Z_L^H$.
2. $Z_H^L - Z_L^L < \frac{1+\sqrt{1+\frac{2r}{\alpha_{opaque}}}}{\sqrt{1+\frac{2r}{\alpha_{opaque}}}-1}$.

Then for any $\alpha \geq 0$, the total ex-ante match value is lower than the ex-ante match value when no secondary market exists.

If Condition #1 does not hold, there is an $\bar{\alpha} \leq \infty$ such that for all $\alpha \in [0, \bar{\alpha}]$, the equilibrium match-value is always lower than the benchmark setting.

Proof: *See Appendix B.*

Suppose types are realized immediately after the primary market ends ($\alpha = \infty$). Condition #1 rules out the situation where the high-tier firm would be happy to poach from the low tier firm even if it knew that the low-tier firm hired the applicant most likely to be of low type. It holds when the interviews at the end of the primary market are successful at identifying low-types (i.e., when $M_{low}(0)$ is small). Importantly, condition #1 implies that when the secondary market is highly transparent, the low-tier firm has the opportunity to hire low-quality workers with high enough probability to deter the high-tier firm from poaching. As a result, unraveling dissipates in highly transparent markets *because* the low-tier firm has an incentive to screen for lower quality workers. This causes a reduction in total match quality relative to the benchmark.

When condition #1 does not hold, extremely transparent secondary markets lead to the high-tier firm choosing to monitor and poach even when the low-tier firm hires a worker that is high type with the minimal probability $M_{low}(0)$. Total match quality is $\approx M_{low}(0)Z_H^H + (1 - M_{low}(0))Z_H^L$, which may or may not be higher than the benchmark.

Condition #2 is important in the moderately transparent range. It excludes "corner cases" where for some values of α, the low-tier firm is ok with hiring high signal workers even though it knows it will be poached with probability 1. These cases arise when the incentives of the firms are aligned with each other. The high-tier firm has a strong desire to screen in the secondary market, *and* the low-tier firm has strong preferences between worker types. Thus, condition #2 is a condition on the preferences of the low-tier firm *in relation* to the high-tier firm's desire to screen. In these moderately transparent regimes, equilibria where the market unravels emerge. Low-tier firms hire high-signal workers but at earlier times, and so the interviews returning these high-signals are not as informative.

In the market design literature, the reductions in match quality that arise in unraveled markets are often seen as the product of the timing of the matches (e.g., Roth and Xing [1994]; Li and Rosen [1998]). If the timing issue is resolved, will total match quality increase?

Theorem 6.4 *Mandating an interviewing and hiring time can reduce the match quality relative to an unraveled market.*

Proof: *See Appendix B.*

Consider a pure-strategy unraveling equilibrium where both firms hire in the primary market: the low-tier firm hires at time $t < 0$ and the high-tier firm hires at $t = 0$. Suppose a third party could ensure that all interviewing in the primary market must occur at $t = 0$. By coordinating the hiring date, one gives more incentives for the high-tier firm to screen. Why? Because the low-tier firm now has access to higher-quality applicants! Since interviews are more informative at $t = 0$, if the low-tier firm hired a high signal worker now, the high-tier firm would want to operate in the secondary market. The threat of poaching is now more serious, and so the low-tier firm has an incentive to hire low-type workers. A mixed strategy equilibrium must exist but with the high-tier firm poaching with non-zero probability. The low-tier firm mixes between hiring a high-signal and low-signal worker. With positive probability, only a single worker is hired by the end of the primary market. This reduces total match quality relative to the pure-strategy unraveled equilibrium that exists without the mandate.

7 Discussion

7.1 Applications

The qualitative characterization of the equilibrium dynamics in my model sheds light on which labor markets may be subject to unraveling. Tables 2 and 3 reproduce Table 1 with the addition of columns describing the transparency of each industry's secondary market. The table below describes the observed characteristics of the labor markets that unravel.

Markets	**Opaque Secondary Market**	**Moderately Transparent Secondary Market**	**Highly Transparent Secondary Market**	**Unraveling**
Corporate Law Associates	X	✓	X	✓
Private Equity Associates	X	✓	X	✓
Investment Banking Analysts	X	✓	X	✓
High-End Chefs and Line Cooks	X	✓	X	✓

Table 2

Why is it reasonable to treat these industries as having "moderately transparent" secondary markets? Consider the world of investment banking. Banks generally have an understanding of the activity of their competitors. For instance, during an IPO of a company, it is publicly known which investment banks are working on the offering. Importantly, banks generally know the specific groups that are working on particular deals. However, it is difficult to observe how much an individual contributed, especially at the analyst and associate levels. Did he merely bring coffee for his bosses, akin to an intern, or was he actively engaged in the deal-structuring process? Similarly, in corporate law, while a high-tier firm can monitor associates at other firms, it is not as easy to assess associate quality compared to a market like that for academic professors, where research is published for public view. Thus, my model predicts that the market for corporate law associates and investment banks will not only unravel, but there will be little poaching in the secondary market in equilibrium. This is consistent with the observation that unraveling in the market has become more extensive, while the lateral movement of associates has decreased substantially over the last few decades.[19]

Now, consider Table 3, which describes the observed characteristics of industries that do not experience unraveling:

Markets	**Opaque Secondary Market**	**Moderately-Transparent Secondary Market**	**Highly Transparent Secondary Market**	**Unraveling**
Hedge Fund Traders	X	X	✓	X
Assistant Professors in Economics	X	X	✓	X
Management Consulting	✓	X	X	X
Programmers and Software Engineers	✓	X	X	X

Table 3

With regard to assistant professors in economics, the secondary market is very transparent, and there is no unraveling. My model highlights that it is not the existence of the centralized system that mitigates unraveling. Instead, such a centralized system is sustainable because of the transparency of the secondary market. On the opposite side of the spectrum is managerial consulting, which has an opaque secondary market. This

[19]See https://www.nalp.org/entry-lateral.

is because casework in consulting is entirely private. Consulting firms are barred from revealing their clients.

Importantly, when there is abundant talent, there are two settings in which labor markets will not unravel. The first is when there is a complete absence of a secondary market (i.e., one that is sufficiently opaque). The other is when there is a secondary market that is sufficiently transparent. Though non-unraveling occurs in both settings, the equilibrium matches are vastly different. In the former, both firms hire at the end of the primary market, while in the latter, there is the type of mixed strategy equilibrium described in Theorem 6.2. Hence, one would expect to see differences in the frequency of junior-level lateral hiring in these industries. Industries with transparent secondary markets will have more lateral hiring than industries with opaque or inactive secondary markets. This is the case when comparing markets for managerial consultants and software engineers to markets for assistant professors in economics and hedge fund traders.

Markets	**Lateral Hiring**
Hedge Fund Traders	✓
Assistant Professors in Economics	✓
Management Consulting	X
Programmers and Software Engineers	X

Table 4: Lateral Hiring in Markets that do not Unravel

It is important to note that my model does not claim that the secondary market's characteristics alone determine whether unraveling occurs or not. Rather, it highlights another avenue by which unraveling can arise. Importantly, it illustrates how unraveling is a phenomenon that is present in markets where firms are *not* worried about whether there will be a shortage of high-quality workers at the end of the primary market. A case where these insights do not apply is the hiring of appellate court judicial clerks. There is no viable secondary market there, yet substantial unraveling occurs. This does not contradict my model. In my model, there is a "short-side" of the market and a "long-side". Unraveling does not occur because the firms are on the long-side. In the judicial clerk market, the size of the viable pool of applicants is not large; firms and applicants fear they are on the long-side of the market. Thus, explanations provided by Niederle et al. (2013) and Ambuehl and Groves (2020) are better suited for this setting.

While the model is described in the context of a labor market, it applies to other two-sided matching markets. For example, in venture capital, one can think of the primary market as the pool of early-stage, pre-seed startups. The secondary market consists of startups that have already received funding and are looking for future series rounds. In sports, the primary market refers to the early-scouting of pre-professional players, while the secondary market refers to professional players' movement across teams.

In venture capital, the firms that find it difficult to earn large returns are typically the smaller, lesser-known ones. It is not that they are unable to find promising startups, but that they are unable to maintain investment relations with the successful startups.[20] More prominent venture firms utilize the smaller ones as screening devices, poaching the "winners" in later series' rounds. As a result, the market has unraveled, with the lesser-known firms investing in startups earlier in their life cycle to prevent dilution.

In sports, the secondary market is transparent because player ability is on public display. My model predicts that not only would little unraveling occur, but mandated "interview dates" (i.e., draft days) would cause inefficient matchings. Low-tier teams (small-market teams) would screen to draft non-star players. However, this is not seen in practice. Is this inconsistent with the model? No. A crucial feature of the model is the worker's freedom to move between firms in the secondary market. Implicitly assumed is that the contracts available to the firms can not prevent mobility. Thus, my model corresponds to a sports league with no restrictive contracts. If players were free to move across teams, inefficient matchings as a result of the adverse informational incentives

[20] I am grateful to Tomasz Tunguz (Partner at Redpoint Ventures) and Aaron Gershenberg (General Partner at Silicon Valley Bank Capital) for this point.

would emerge (Rottenberg [1956]; El-Hodiri and Quirk [1971]). The reason such inefficiencies are not observed is due to the existence of contracts preventing mobility. Therefore, teams no longer need to be as concerned with players being poached.[21]

7.2 Relation to Innovation

Theorems 6.3 and 6.4 highlight a general phenomenon regarding markets with mobile assets of unknown quality. Within the labor market context, time is the crucial dimension that affects the ability to screen the assets (i.e., workers). However, in markets involving innovation, effective screening may be contingent on costly investment.

Prospective employees are analogous to "potential ideas" that companies can screen and choose to develop. The low-tier firm, F_L, corresponds to an entrant in the market, while the high-tier firm, F_H, corresponds to an incumbent. Conditional on "matching" (selecting a potential innovation to develop), the innovation generates profits for the company. Competitors can observe informative signals regarding the quality of the innovation and make a "poaching decision", which corresponds to developing a substitute themselves. This would reduce the profits of the innovator.

The threat of copycat innovation is particularly detrimental to the entrant. As a result, an informative secondary market discourages investment in the screening of potential ideas. Furthermore, interpreting Theorem 6.4 through this lens demonstrates that the entrant will develop most innovations. This is consistent with the observation that incumbents are less likely to develop innovations compared to entrants (e.g., Bresnahan et al. [2012]; Awaya and Krishna [2020]).

Example 1 *Time is continuous from $[0,\infty]$, and there is a set of N ideas. Time $t=0$ represents the "primary market stage", and $(0,\infty)$ represents the "secondary market stage". Each idea has i.i.d probability β of being turned into a novel innovation (high type); otherwise, it becomes an average innovation (low type). At $t=0$, firms exert effort $e \geq 0$ to screen ideas. Screening is modeled by a function M as in Section 2.2,*

[21] Under free agency rules, such movement can not be prohibited indefinitely (i.e., the restrictive contracts only last for a fixed number of years). However, leagues such as the NBA have implemented rules that allow teams to pay their players on expiring contracts significantly more than any competitor.

except that it is a function of effort rather than time. Effort is costly, represented by a convex cost function $c(e)$. The flow payoff for firm F_i with an innovation of type θ is Z^i_θ. These payoffs have the same structure as in Section 2.3.[22]

Once the idea is selected, the innovation is realized, and the secondary market stage begins. A public signal regarding the innovation's quality is observed:

$$d\pi_t = \mu_\theta dt + \sigma dB_t$$

This description is analogous to a specific instance of my model. While there is no time dimension in the primary market stage, the existence of an effort cost indirectly caps the firms' screening ability at $t = 0$. Therefore, effort operates in the same way as time-selection does in my model. At a technical level, the solution to the innovation game is equivalent to the equilibrium found under a mandated hiring time (Theorem 6.4). Thus, the unique equilibrium of the innovation game has the low-tier firm choosing an effort level and a non-unit probability of selecting a high-signal idea. The high-tier firm mixes between poaching (exerting no effort in the primary market) and screening at the optimal effort level. This equilibrium is inefficient relative to the setting with no secondary market (i.e., a setting with long-lasting patents).

7.3 Potential Policy Solutions

Two crucial features of the model are the low-tier firm's ability to block off information in the primary market once it matches with a worker and the freedom of the worker to move between firms in the secondary market. Hence, two interventions may mitigate unraveling and increase efficiency:

1. Improving the Flow of Information in the Primary Market.

2. Controlling Mobility in the Secondary Market.

Concerning the former, the growth of the internet has facilitated communication; websites such as LinkedIn and Github have made for more transparency. While these

[22]The negative payoff to the high-tier firm from implementing an average innovation represents opportunity and reputational cost.

could alleviate unraveling in the primary market, they also improve monitoring ability in the secondary market. This can actually increase inefficiency due to the findings in Theorems 6.3 and 6.4.

Labor mobility is generally thought of as a positive, and there are many examples where restricting mobility leads to inefficiencies. My paper points to an inefficiency caused by strategic responses to mobility: unraveling and reduced screening. In professional sports, for instance, where leagues desire a balanced distribution of talent, incentivizing teams to screen effectively will be difficult if there is no restriction on player movement. In areas of innovation, patents play this role. In corporate law, non-competes are utilized, but those have been difficult to enforce, and there is no equivalent contract for associate-level positions. Matching markets with an abundance of talent and an active secondary market may benefit from long-term, restrictive contracts.

8 Robustness: Multiple Firms

The model of unraveling developed in this paper captures the rich informational incentives at play. The assumption of only two firms existing in the market allows for a complete closed-form characterization of the informational dynamics. The model is robust to changes in the number of firms. Allowing for multiple firms does not change the qualitative features of the results above. However, it does elucidate the importance of the *risk of poaching* to unraveling. The cost of adding multiple firms to the model is the loss of closed-form solutions. I will be informal in the definition of strategies in this section for ease of exposition.

To illustrate the incentives, consider an environment where N_L low-tier firms are hiring in the primary market and N_H high-tier firms are specializing in secondary market hiring, with each firm being allowed to monitor al low-tier firms. I assume that the high-tier firms receive the same signals in the secondary market.

Example 2 *Consider two low-tier firms, F_{L_1} and F_{L_2}, and a high-tier firm F_H. The high-tier firm commits to hiring laterally, observing two processes, one for each worker:*

$$d\pi^{(1)} = \mu_\theta dt + dB_t^{(1)} \text{ and } d\pi^{(2)} = \hat{\mu}_\theta dt + dB_t^{(2)}$$

Keeping with the usual notation, $p_t^{(1)}$ and $p_t^{(2)}$ will denote the respective posterior beliefs about each worker at time t, given the observations of the signal processes. If F_H is committed to hiring on the secondary market, its poaching problem is now:

$$\max_\tau \mathbb{E}\left[e^{-r\tau} \max_{i\in\{1,2\}} \left\{p_\tau^{(i)}(Z_H^H - Z_L^H) + Z_L^H\right\} | \mathscr{F}_t^{\pi^{(1)}}, \mathscr{F}_t^{\pi^{(2)}}\right]$$

The optimal stopping problem is much more complex, as F_H must keep track of its beliefs about both workers. At any given point in time, the worker whom F_H is more pessimistic about serves as an endogenous outside option. Hence, the boundary of the continuation region (e.g., poaching threshold belief) is increasing in the minimum of the beliefs.

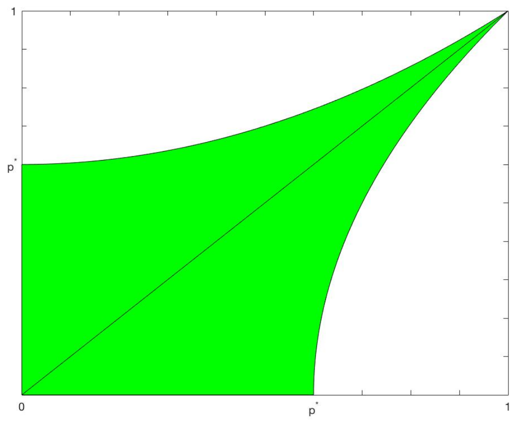


Figure 1: *Optimal continuation region in belief space. The quantity p^* corresponds to the threshold in the one-dimensional setting when there is one high-tier firm monitoring a single low-tier firm.*

When it monitors two low-tier firms, the payoff to the high-tier firm is higher than when it only monitors a single one. However, from the low-tier firms' perspective, the individual probability of being poached has declined!

The example highlights the important idea of the credibility of poaching. In the model with only a single low-tier and high-tier firm, poaching is essentially guaranteed to occur if the high-tier firm operates in the secondary market. If another low-tier firm is added, the high-tier firm's payoff from poaching increases because of optionality. However, that same optionality reduces the threat of poaching as viewed by each low-tier firm. To generalize this intuition:

Proposition 8.1 *There exists* $J, K, W > 0$ *such that for* $\frac{N_L}{N_H} > J$*,* $N_L - N_H > K$*, and* $N > W$*, there is no unraveling.*

Proof: *See Appendix B.*

If there are very few firms poaching relative to the number of firms available to poach from, then poaching is not a serious threat. When the individual threat of poaching reduces to approximately 0, and there is no shortage of talent, the market will not unravel. Moreover, the lack of a threat of poaching eliminates the adverse signaling incentives, allowing the ex-ante efficient matching to be achieved.

The theorem highlights the idea that in industries with a secondary market, unraveling is a phenomenon that occurs when there is a hierarchy of firms, and poaching is a credible event. Some well-known markets that fit this description are the hedge fund space, private equity, academia, corporate law, venture capital, and professional sports.

9 Conclusion

In most industries, the initial match between an employee and a firm is not permanent. After a worker is hired, it is often the case that she will receive offers from competing firms. This phenomenon occurs in numerous industries, ranging from academia to investment banking to corporate litigation. The addition of this secondary market, whereby firms can poach workers from other firms, introduces a new channel by which unraveling can occur. Unraveling is no longer a race to acquire top talent but a strategic decision made by low-tier firms in an effort to keep the worker it does hire. However, even with an active and fully transparent secondary market, there may still be adverse effects. While transparency decreases unraveling, it does so at the expense of efficiency. A highly transparent secondary market incentivizes the low-tier firm to screen workers in order to ensure that it has not hired a high-quality worker.

A Strategies and Equilibrium

A.1 Definitions

Conceptually, the strategies available to the firms are intuitive. Given F_H will never enter the primary market before $t = 0$, its decision is whether to operate in the secondary market or the primary market at $t = 0$. It can make this choice based on the time at which F_L chooses to hire. To formalize this requires additional care. Endow the interval $\mathscr{E} = [-T, 0]$ with the Borel Σ-algebra $\mathscr{B}$. For rigor, we also define the set $E = [-T, 0]$ and endow E with the Σ-algebra $\mathscr{B}$ as well. I refer to $\mathscr{E}$ as the sample space and E the outcome space.[23]

Definition A.1 *A* **hiring policy** *adapted to a set* $\hat{\mathscr{E}} \subset \mathscr{E}$*, is a function* $h : \hat{\mathscr{E}} \longrightarrow [0, 1]$.

The function $h(t)$ represents the probability of choosing a high-signal worker when choosing to interview and hire at any time $t \in \hat{\mathscr{E}}$.

Definition A.2 *A* **primary-market strategy** *is a probability measure* μ *on* $(\mathscr{E}, \mathscr{B})$ *and a hiring policy* h *adapted to* $supp(\mu)$.

F_L chooses a probability measure over $([-T, 0], \mathscr{B})$. The random variable $X_\mu : \mathscr{E} \longrightarrow E$, where $X_\mu(\omega) = \omega$, has distribution F_μ. The realization of X is the time at which F_L enters the primary market. Upon observing the realization $x \in E$, F_H formulates beliefs about whether F_L hired out of the high-pool or low-pool.

Definition A.3 *A* **belief mapping** $f : E \longrightarrow [0, 1]$ *represents the probability* F_H *attaches to* F_L *hiring a worker out of the high-pool, conditional on the time* $x \in E$ *at which* F_L *entered the primary market.*

Conceptually, a strategy for F_H is a decision based on the observed history and its belief about the worker hired by F_L. The strategy will dictate whether F_H hires on the primary market, and the poaching rule it will use conditional on operating in the secondary market. If F_H has belief p_0 about the worker hired by F_L, let $ST(p_0)$ denote the set of all possible poaching rules.

[23] See Karatzas and Shreve (1998) for definitions of the mathematical terms.

Definition A.4 *A* **strategy for** F_H *is a mapping* $W : E \times [0,1] \longrightarrow [0,1] \times ST$ *from observations and beliefs to a probability of operating in the primary market and a poaching rule conditional on operating on the secondary market. In other words,* $W(x,g) = (m,\tau)$ *where* $m \in [0,1]$ *and* $\tau \in ST(g)$.

Given there is no commitment, F_H chooses whether to operate on the primary market at $t = 0$ or on the secondary market using poaching rule τ.

Definition A.5 *An* **equilibrium** *is a primary-market strategy* (μ,h) *for* F_L *and a strategy-belief pair* (W,f) *for* F_H*, such that:*

1. *Each firm is best-responding at each information set.*
2. F_H*'s beliefs are consistent on the support of* μ.

The equilibrium concept used is Perfect Bayesian Equilibrium. Since the strategy for F_H is to choose whether to operate on the secondary market or the primary market, it follows from Hendon et al. (1996) that any perfect bayesian equilibrium will also be a sequential equilibrium.

Proof of Lemma 2.1:

Since N is discrete, there is a technicality that must be considered. There is a non-zero probability that all individuals emit the same signal in an interview. That is, after interviewing, the firm sees only low-signals or only high-signals. In this situation, I assume that the firm randomizes between whom it hires. Recognize that the probability of this event occurring tends to 0 rapidly (e.g., converges exponentially). Given the primary market's informational structure M, consider the corresponding mapping from time to probabilistic binary tests.

I will show that all workers will accept any offer they receive in the primary market when N is sufficiently large. Intuitively, for N small, receiving an offer provides the worker with more information about her type. She may strategically reject because she now believes she has a better chance of receiving an offer from the high-tier firm. For N sufficiently large, there is no gain from such "strategic rejection".

Define the function $\hat{\beta}(t,N)$ as the posterior probability that a worker is of type $\theta = H$ conditional on receiving an offer at time t when there are N workers available. Let $\Delta_{t,\beta} = \beta(1-x_H^t) + (1-\beta)(1-x_L^t)$ denote the ex-ante probability of failing the test at time t when the probability a worker is of type $\theta = H$ is β. It follows that:

1. $\hat{\beta}(t,N+1) = (N+1)\beta \cdot \left(x_H \sum_{k=0}^{N} \binom{N}{k} \frac{\Delta_{t,\beta}^{N-k}(1-\Delta_t)^k}{k+1} + \frac{(1-x_H)\Delta_{t,\beta}^N}{N+1}\right)$
2. $\hat{\beta}$ is weakly increasing in t and N
3. $\lim_{N\to\infty} \hat{\beta}(t,N) = \frac{\beta x_H}{\beta x_H + (1-\beta)x_L}$

Suppose there are $N+1$ potential workers, and all workers besides worker i accept any offer. From worker i's perspective, if she received an offer from F_L at time t and rejected it, her expected payoff is the probability of receiving an offer at time 0 from the high-tier firm multiplied by δ. Since δ is a constant and normalize it to 1.

$$\text{Payoff from Rejection} = \left(1-\Delta_{t,\hat{\beta}(t,N+1)}\right) \sum_{k=0}^{N-1} \binom{N-1}{k} \frac{\Delta_{t,\beta}^{N-k-1}(1-\Delta_{t,\beta})^k}{k+1} + \frac{\Delta_{t,\hat{\beta}(t,N+1)}\Delta_t^{N-1}}{N}$$

$$< \sum_{k=0}^{N-1} \binom{N-1}{k} \frac{\Delta_{t,\beta}^{N-k-1}(1-\Delta_{t,\beta})^k}{k+1} + \frac{1}{N}$$

$$< \sum_{k=0}^{N-1} \binom{N-1}{k} \frac{\Delta_{t,\beta}^{N-k-1}(1-\Delta_{t,\beta})^k}{k+1} + \frac{1}{N}$$

$$\text{For large } N \implies < N \binom{N-1}{\frac{N-1}{2}} \frac{2^{1-N}}{\frac{N-1}{2}+1} + \frac{1}{N}$$

Using Stirling's formula, the expression above approaches 0 as N grows large.

■

B Proofs

For notational convenience and ease of exposition, I define the following terms:

$$p_{max} = M_{high}(0) \text{ and } p_{min} = M_{low}(0)$$

$$R_1 = \frac{1 - \sqrt{1 + \frac{2r}{\alpha}}}{2}$$

$$R_2 = \frac{1 + \sqrt{1 + \frac{2r}{\alpha}}}{2}$$

$$\widetilde{Z}_\theta^H = Z_\theta^H - c(w)$$

$$d^H = -\frac{Z_H^H}{Z_L^H} \qquad \tilde{d}^{F_H} = -\frac{\widetilde{Z}_H^H}{\widetilde{Z}_L^H} \qquad d^L = \frac{Z_H^L}{Z_L^L}$$

For computational convenience, I will sometimes work in the log-odds space of the beliefs, $Q = log(\frac{p}{1-p})$. I refer to both Q and p as the "belief" due to this isomorphism.

Define $\Pi_i(p,w) = pZ_H^i + (1-p)Z_L^i - w = \frac{1}{1+e^Q}(e^Q Z_H^i + Z_L^i) - w$ to be the expected match value less the wage from hiring worker (Q,w) conditional on employing the worker forever.

B.1 Optimal Poaching

Lemma B.1 *The belief process $\{p_t\}$ has the strong markov property.*

Proof:

Since $\{\pi_t\}$ is Markovian, p_t depends only on the value of π_t. Bayes' rule yields:

$$p_t = \frac{p_0 f_t(\pi_t|\theta = H)}{p_0 f_t(\pi_t|\theta = H) + (1-p_0) f_t(\pi_t|\theta = L)}$$

Using Ito's Lemma and the Innovation Theorem:

$$dp_t = \frac{2\bar{\mu}}{\sigma}(1-p_t)p_t d\hat{B}_t$$

$$\text{where } \hat{B}_t = \frac{1}{\sigma}(\pi_t - 2\bar{\mu}\int_0^t p_s ds) \text{ is the innovation process}$$

The innovation theorem implies that the innovation process $\hat{B}_t$ is a Brownian motion with respect to the filtration $\{\mathscr{F}^{\pi_t}\}$. The lemma follows.

■

Lemma B.2 *Given a Markov process $\{x_t\}$ and a continuous function g, consider the optimal stopping problem:*

$$\sup_\tau \mathbb{E}[e^{-r\tau} g(x_t)|x_0]$$

There exists a solution of the form $\tau = \inf\{t|x_t \notin (a,b)\}$.

Proof: Given there is exponential discounting and the process $\{x_t\}$ is Markov, the value function V takes the form:

$$V(x_0) = \sup_\tau \mathbb{E}[e^{-r\tau} g(x_t)|x_0]$$

By standard arguments, the continuation region is given by $C = \{x : V(x) > g(x)\}$ and the stopping region by $S = \{V(x) = g(x)\}$. Due to the continuity of the process x_t, I can restrict attention to the *connected* subset $\hat{C} \subset C$ around x_0. This continuation region provides the same expected value. Therefore, there is an optimal stopping time of the desired form.

■

Proof of Proposition 4.1:

Using a standard change of variables, define the log-odds ratio $Q_t = log(\frac{p_t}{1-p_t})$. Applying Bayes' rule yields:

$$Q_t = log\Big(\frac{p_0}{1-p_0}\Big) + log\Big(\frac{f_t(\pi_t|\theta=H)}{f_t(\pi_t|\theta=L)}\Big)$$

$$\implies Q_t = Q_0 + \frac{\mu_H - \mu_L}{\sigma^2}\pi_t + \frac{\mu_L^2 - \mu_H^2}{2\sigma^2}t$$
$$\implies dQ_t = \frac{(\mu_H - \mu_L)([\mu_\theta - \mu_L] + [\mu_\theta - \mu_H])}{2\sigma^2}dt + \frac{\mu_H - \mu_L}{\sigma}B_t$$

From Peskir and Shiryaev (2006), it follows that Q_t has the strong Markov property. By Lemma B.2, the optimal poaching rule τ is characterized by a continuation region around Q_0. There is no cost associated with observation, which implies that there is no "rejection" threshold. Thus, the continuation region is of the form $(-\infty, B)$. It follows that the optimal poaching rule for F_H is a stopping time τ of the form $\tau = \inf\{t \geq 0 : Q_t \geq B\}$, for some $B > 0$.

Given a worker at F_L with wage w and probability Q_0 of being high type, let $\Gamma_H(Q_0, w, \tau)$ denote the payoff to F_H from following τ. I first explicitly compute $\Gamma_H(Q_0, w, \tau)$, and then maximize it over all threshold stopping times.

Consider a poaching rule where F_H hires the worker if its belief about the worker reaches a threshold B, and commits to never hiring once beliefs fall below b. Such a poaching rule can be represented by the stopping time $\tau = \inf\{t \geq 0 : Q_t \notin (b, B)\}$. With initial condition $Q_0 \in (b, B)$, it follows:

$$Pr(Q_\tau = B|\theta = H)\mathbb{E}[e^{-r\tau}|\theta = H, Q_\tau = B] = \frac{e^{-R_1(Q_0-b)} - e^{-R_2(Q_0-b)}}{e^{-R_1(B-b)} - e^{-R_2(B-b)}} = \xi(Q_0, b, B)$$

$$Pr(Q_\tau = B|\theta = L)\mathbb{E}[e^{-r\tau}|\theta = L, Q_\tau = B] = \frac{e^{-R_1(Q_0-b)} - e^{-R_2(Q_0-b)}}{e^{-R_1(B-b)} - e^{-R_2(B-b)}} = \xi(Q_0, b, B)e^{Q_0 - B}$$

Since the optimal poaching rule has $b = -\infty$, taking limits shows that the payoff under a stopping time of the form $\tau = \inf\{t \geq 0 : Q_t \geq B\}$ with initial condition Q_0 is precisely:

$$\Gamma_H(Q_0, w) = \tilde{Z}_H^H \frac{e^{Q_0}}{1 + e^{Q_0}} e^{R_1(B - Q_0)} + \frac{Z_L^H}{1 + e^{Q_0}} e^{R_2(Q_0 - B)}$$

While this function is not concave, it attains its maximum in the interior[24], so taking first order conditions to calculate the optimal threshold B^*, yields:

$$\implies B^* = -log\left(\frac{\widetilde{Z}_H^H R_1}{\widetilde{Z}_L^H R_2}\right)$$

Thus, the optimal poaching rule is:

$$\tau^* = \inf\left\{t \geq 0 : Q_t \geq -log\Big(\frac{\widetilde{Z}_H^H R_1}{\widetilde{Z}_L^H R_2}\Big)\right\} = \inf\left\{t \geq 0 : p_t \geq \frac{\widetilde{Z}_L^H R_2}{\widetilde{Z}_H^H R_1 + \widetilde{Z}_L^H R_2}\right\}$$

■

B.2 Payoff Functions

Note the following. As the high-tier firm's payoff function depends on its belief, consistency is assumed. For the low-tier firm, the probability that the worker is of high-type depends on its action choice. The payoff to the low-tier firm depends on this as well as the high-tier firm's belief.

$$\Gamma_H(\tilde{p}, w) = \tilde{p}(Z_H^H - w)\xi(\tilde{Q}, B^*) + (1-\tilde{p})(Z_L^H - w)\xi(\tilde{Q}, B^*)e^{-(B^*-\tilde{Q})}$$

$$\Sigma_L(p, \tilde{p}, w) = p(Z_H^L - w)\Big(1 - \xi(\tilde{Q}, B^*)\Big) + (1-p)(Z_L^L - w)\Big(1 - \xi(\tilde{Q}, B^*)e^{-(B^*-\tilde{Q})}\Big)$$

Lemma B.3 *Suppose F_H operates in the secondary market using poaching rule τ^*. The payoff to F_L from employing a worker under initial belief Q_0 and wage w is:*

$$\Gamma_L(Q_0, w) = \underbrace{\frac{1}{1+e^{Q_0}}(e^{Q_0}(Z_H^L - w) + (Z_L^L - w))}_{\text{Expected Productivity}} - \underbrace{\frac{e^{R_2(Q_0-B^*)}}{1+e^{Q_0}}\left[(Z_H^L - w)\frac{\widetilde{Z}_L^H R_2}{\widetilde{Z}_H^H R_1} + (Z_L^L - w)\right]}_{\substack{\text{Loss due to possibility} \\ \text{of losing worker to } F_H}}$$

[24] As $B \longrightarrow -\infty$, it approaches $-\infty$. As $B \longrightarrow \infty$, it approaches 0. Finally, due to the restrictions on k, there exists B such that the expression is positive. Thus, a global maximum is attained in the interior.

Proof:

Proposition 4.1 implies that F_H will use the stopping rule τ^*. The payoff to F_L from employing a worker at wage w with probability Q_0 of being of a high type is:

$$\Gamma_L(Q_0,w) = p_0(Z_H^L - w)Pr(Q_{\tau^*} = B^*|\theta = H)(1-\mathbb{E}[e^{-r\tau^*}|\theta = H, Q_{\tau^*} = B^*])$$

$$+(1-p_0)(Z_L^L - w)\left[Pr(Q_{\tau^*} = B^*|\theta = L)(1-\mathbb{E}[e^{-r\tau^*}|\theta = L, Q_{\tau^*} = B^*]) + 1 - Pr(Q_{\tau^*} = B^*|\theta = L)\right]$$

$$\Longrightarrow \Gamma_L(Q_0,w) = p_0(Z_H^L - w)(1-\xi(Q_0,-\infty,B^*)) + (1-p_0)(Z_L^L - w)(1-\xi(Q_0,-\infty,B^*)e^{Q_0-B^*})$$

$$= p_0(Z_H^L - w)(1-e^{R_1(B^*-Q_0)}) + (1-p_0)(Z_L^L - w)(1-e^{Q_0-B^*}e^{R_1(B^*-Q_0)})$$

$$= p_0(Z_H^L - w)(1-e^{R_1(B^*-Q_0)}) + (1-p_0)(Z_L^L - w)(1-e^{R_2(Q_0-B^*)})$$

$$= \frac{1}{1+e^{Q_0}}(e^{Q_0}(Z_H^L - w) + (Z_L^L - w)) - \frac{e^{R_2(Q_0-B^*)}}{1+e^{Q_0}}\left[(Z_H^L - w)e^{B^*} + (Z_L^L - w)\right]$$

$$\underbrace{\frac{1}{1+e^{Q_0}}(e^{Q_0}(Z_H^L - w) + (Z_L^L - w))}_{\text{Expected Productivity}} - \underbrace{\frac{e^{R_2(Q_0-B^*)}}{1+e^{Q_0}}\left[(Z_H^L - w)\frac{\widetilde{Z}_L^H R_2}{\widetilde{Z}_H^H R_1} + (Z_L^L - w)\right]}_{\substack{\text{Loss due to possibility} \\ \text{of losing worker to } F_H}}$$

■

Lemma B.4 *For any p_0 and $w \geq 0$, there exists p_0' such that $\Gamma_L(p_0',0) \geq \Gamma_L(p_0,w)$.*

Proof:

Conditional on worker type and F_H using a threshold poaching rule, the expected discounted probabilities of being poached are:

$$Pr(Q_\tau = B|\theta = H)\mathbb{E}[e^{-r\tau}|\theta = H, Q_\tau = B] = \lim_{b\to-\infty}\xi(Q_0,b,B) = e^{R_1(B-Q_0)}$$

$$Pr(Q_\tau = B|\theta = L)\mathbb{E}[e^{-r\tau}|\theta = L, Q_\tau = B] = \lim_{b\to-\infty}\xi(Q_0,b,B)e^{Q_0-B} = e^{-R_2(B-Q_0)}$$

Recognize that each of the above quantities depends on the *difference* between the threshold belief and the initial belief that the worker is of high type.

Consider a worker that is of high type with probability Q_0. Let $B^*(w)$ denote the poaching threshold when F_L pays the worker a wage $w \geq 0$. Given $\hat{Q}_0 < Q_0$, let $w' > 0$ be such that $Q_0 - \hat{Q}_0 = B^*(w') - B^*(0)$. I will show that $\Gamma_L(\hat{Q}_0, 0) \geq \Gamma_L(Q_0, w')$.

Because the expected discounted probabilities of being poached are the same for worker $(\hat{Q}, 0)$ and (Q_0, w'), it is sufficient to prove:

$$\Pi(Q_0,0) - \Pi(\hat{Q}_0,0) < \Pi(Q_0,0) - \Pi(Q_0,w')) = w'$$

Letting $\Delta = Q_0 - \hat{Q}_0 > 0 \Longrightarrow$:

$$\Pi(Q_0,0) - \Pi(\hat{Q}_0,0) = \frac{e^{Q_0}}{1+e^{Q}} \cdot \frac{1-e^{-\Delta}}{1+e^{Q_0-\Delta}} \cdot (Z_H^L - Z_L^L)$$

$$= \frac{e^{Q_0}}{1+e^{Q}} \cdot (1-e^{\Delta}) \cdot \frac{-e^{-\Delta}}{1+e^{Q_0-\Delta}} \cdot (Z_H^L - Z_L^L)$$

$$\leq \frac{-e^{B^*(0)}}{1+e^{B^*(0)}} \cdot \frac{(1-e^{\Delta})}{1+e^{B^*(0)-\Delta}} \cdot (Z_H^L - Z_L^L)$$

$$= -(1-e^{\Delta}) \cdot \frac{R_1 R_2 Z_H^H Z_L^H}{(R_1 Z_H^H + R_2 Z_L^H)(R_1 Z_H^H + R_2 Z_L^H e^{\Delta})} \cdot (Z_H^L - Z_L^L)$$

$$< (1-e^{\Delta}) \cdot \frac{Z_H^H Z_L^H}{(Z_H^H - Z_L^H)(Z_H^H - Z_L^H e^{-\Delta})} \cdot (Z_H^L - Z_L^L)$$

$$< (1-e^{\Delta}) \cdot \frac{Z_H^H Z_L^H}{Z_H^H - Z_L^H e^{-\Delta}}$$

Since $B^*(w') - B^*(0) = \Delta$ it must be that $w' \geq \dfrac{Z_H^H Z_L^H (1-e^{\Delta})}{Z_H^H - Z_L^H e^{\Delta}}$.

$$\Longrightarrow \Pi(Q_0,0) - \Pi(\hat{Q}_0,0) < (1-e^{\Delta}) \cdot \frac{Z_H^H Z_L^H}{Z_H^H - Z_L^H e^{-\Delta}} \leq w'$$

■

Lemma B.5 $\Gamma_L(p_0, 0)$ *is single-peaked in* p_0.

Proof:

With the wage set at 0, Γ_L can be viewed as a function of a single variable, p_0. As before, I will use the change of variables $Q = log(\frac{p}{1-p})$ for algebraic convenience. Taking the expression for Γ_L in Lemma B.3, I can derive the following closed form expression for the derivative of $\Gamma_L(Q)$:

$$\frac{\partial \Gamma_L}{\partial Q} = \frac{e^Q}{(1+e^Q)^2}(Z_H^L - Z_L^L) - e^{R_2 Q}\left(\frac{R_2 + R_2 e^Q - e^Q}{(1+e^Q)^2}\right)\left(-\tilde{d}^{F_H} \cdot \frac{R_1}{R_2}\right)^{R_2 - 1}\left(Z_H^L - Z_L^L \frac{R_1}{R_2}\tilde{d}^{F_H}\right)$$

Therefore, $\Gamma_L(Q)$ is decreasing in Q whenever:

$$e^Q(Z_H^L - Z_L^L) - e^{R_2 Q}\left(R_2 - R_1 e^Q\right)\left(-\tilde{d}^{F_H} \cdot \frac{R_1}{R_2}\right)^{R_2 - 1}\left(Z_H^L - Z_L^L \frac{R_1}{R_2}\tilde{d}^{F_H}\right) < 0$$

$$\iff -1 + d^L - e^{(R_2-1)Q}\left(R_2 - R_1 e^Q\right)\left(-\tilde{d}^{F_H} \cdot \frac{R_1}{R_2}\right)^{R_2 - 1}\left(d^L - \frac{R_1}{R_2}\tilde{d}^{F_H}\right) < 0$$

$$\iff (R_2 - 1)Q + log(R_2 - R_1 e^Q) + (R_2 - 1)log(\tilde{d}^{F_H}) + (R_2 - 1)log\left(-\frac{R_1}{R_2}\right) + log\left(d^L - \frac{R_1}{R_2}\tilde{d}^{F_H}\right) > log(d^L - 1)$$

Notice that the left-hand side is increasing in Q, holding everything else fixed. Therefore, Γ_L is single-peaked in Q for $Q \leq B^*$.

■

Proof of Proposition 5.1:

Since the constrained function $\Gamma_L(p_0, 0))$ is single-peaked in p_0, there is a unique $p_0^* = \arg\max_{p_0 \in [0,1]} \Gamma_L(p_0, 0))$. By Lemma B.4, this is the optimum of the unconstrained function $\Gamma_L(p_0, w)$.

■

Lemma B.6 ***Comparative Statics***

1. $\lim_{\alpha \to 0} Q^* = B^*$
2. $\lim_{\alpha \to \infty} Q^* = -\infty$
3. $\lim_{d^L \to \infty} Q^* = K(\alpha, \tilde{d}^H)$ *for some constant* $K(\alpha, \tilde{d}^H)$

Proof:

This follows immediately from the first-order condition identified in the proof of Lemma B.5. ■

Proof of Proposition 5.2:

Let $\bar{\alpha} = p^{*-1}(\beta)$. For any $\alpha \leq \bar{\alpha}$ there exists a time t^* such that $M_{high}(t^*) = p^*(\alpha)$. By Proposition 5.1, it is optimal for the low-tier firm to hire the high signal worker at time t^*, offering a wage of 0.

For $\alpha > \bar{\alpha}$, it follows that $p^*(\alpha) < M_{high}(M_{low}^{-1}(p^*))$. There is no equilibrium where the high-tier firm can believe that the low-tier firm is hiring a worker with probability p^* of being high type. Otherwise, at the time of hiring, the low-tier firm would deviate to hiring the high-signal worker. By Lemma B.8 in the following section, there is a probability $p_{ind} \geq p^*$ such that if the high-tier believed the worker hired by the low-tier firm was high type with probability p_{ind}, the low-tier firm would be indifferent between worker types.[25]

Consider the following constrained game, where the low-tier firm must hire at time t. In the constrained game, the high-tier firm's belief must be that at a wage of w, the low-tier firm hires a worker with probability $\min\{p_{ind}(w), M_{high}(t)\}$ of being high type.

Define $\hat{w}(t) = \max_w \Gamma_L(M_{high}(t), M_{high}(t), w)$ and consider the functions $\chi(t)$ and $w^*(t)$ constructed as follows:

1. If $\Gamma_L\Big(M_{high}(t), M_{high}(t), \hat{w}(t)\Big) > \Gamma_L(p_{ind}, 0)$, then the low-tier firm hires the high-signal worker at wage $\hat{w}(t)$. Set $w^*(t) = \hat{w}(t)$ in this case. Set $\chi(t) = 1$.

[25]Note: p_{ind} depends on the offered wage.

2. If $\Gamma_L\Big(M_{high}(t), M_{high}(t), \hat{w}(t)\Big) \leq \Gamma_L(p_{ind}, 0)$, the low-tier firm pays wage 0 and mixes between hiring high and low-signal workers to induce a belief $p_{ind}(0)$ about the worker. Set $w^*(t) = 0$ and $\chi(t)$ equal to the probability with which the low-tier firm hires a high-signal worker.

The functions $\chi(t)$ and $w^*(t)$ constitute a strategy for the low-tier firm. To demonstrate that it is optimal in the constrained game, I show that the low-tier firm would never choose a wage w to induce a belief $p = p_{ind}(w) < M_{high}(t)$. Suppose otherwise. The payoff to the low-tier firm would be $\Sigma_L(p, p, w)$. Since the low-tier firm is indifferent between worker types:

$$\Sigma_L(p,p,w) = \Sigma_L(p_{ind}(0), p_{ind}(w), w) < \Sigma_L(p_{ind}(0), p_{ind}(0), w) < \Sigma_L(p_{ind}(0), p_{ind}(0), 0)$$

Hence, the low-tier firm will never choose to induce a belief $p_{ind}(w) \in (p_{ind}(0), M_{high}(t))$. The strategy is indeed optimal. In an abuse of notation, denote the payoff from the optimal strategy in the constrained game at time t as $\Gamma_L(\chi(t), w^*(t))$.

Returning to the unconstrained game, the low-tier firm's equilibrium strategy requires selecting the times t with the maximum constrained game payoffs. Let D denote the set of times such that:

$$D = argmax_t \Gamma_L(\chi, w^*)$$

Any equilibrium consists of the low-tier firm choosing a distribution over times $t \in D$ and following $\chi(t)$ and $w^*(t)$.

NOTE 1 *In the edge case where $p_{ind} < M_{low}(0)$, the low-tier firm will hire a worker that is high type with probability $M_{low}(0)$. The high-tier firm's belief is obviously $M_{low}(0)$.*

■

Lemma B.7 *Suppose F_L hires a worker of type Q_0 at wage w . F_H prefers to hire in the primary market if and only if:*

$$\underbrace{\frac{e^{Q_{max}}}{1+e^{Q_{max}}} Z_H^H + \frac{1}{1+e^{Q_{max}}} Z_L^H}_{\text{Payoff from hiring in the Primary Market}} \geq \underbrace{\frac{e^{R_2 Q_0}}{1+e^{Q_0}} \left(\frac{\alpha-1}{\alpha+1} \cdot \frac{-\widetilde{Z}_H^H}{\widetilde{Z}_L^H} \right)^{R_2-1} \frac{\widetilde{Z}_H^H}{R_2}}_{\Gamma_H(Q_0, w)}$$

Proof:

Assuming that the worker employed by F_L will have initial secondary market prior $Q_0 < B^*$, F_H's payoff from monitoring is:

$$\Gamma_H(Q_0, w) = \widetilde{Z}_H^H \frac{e^{Q_0}}{1+e^{Q_0}} e^{R_1(B^*-Q_0)} + \frac{\widetilde{Z}_L^H}{1+e^{Q_0}} e^{R_2(Q_0-B^*)}$$

$$= \frac{e^{R_2 Q_0}}{1+e^{Q_0}} \left(\frac{\alpha-1}{\alpha+1}\tilde{d}^{F_H}\right)^{R_2-1} \left(\widetilde{Z}_H^H - \widetilde{Z}_L^H \tilde{d}^{F_H} \frac{R_1}{R_2}\right)$$

$$= \frac{e^{R_2 Q_0}}{1+e^{Q_0}} \left(\frac{\alpha-1}{\alpha+1}\tilde{d}^{F_H}\right)^{R_2-1} \frac{\widetilde{Z}_H^H}{R_2}$$

■

B.3 Indifference Beliefs

When is the low-tier firm indifferent between the type of worker it hires? Similarly, when is the high-tier firm indifferent between poaching on the secondary market and hiring at the end of the primary market?

Define p_{ind} and $\bar{p}$ to be these beliefs, respectively:

$$\Sigma_L(p, p_{ind}, w) = \Sigma(p', p_{ind}, w) \text{ for all } p, p' \in (0,1)$$

$$\Gamma_H(\bar{p}, w) = \Pi_H(p_{max}, w)$$

NOTE 2 *The indifference beliefs are endogenous and depend crucially on α, the wage w, the match quality values, and sorting ability at the end of the primary market (p_{max}). For notational convenience, I will suppress dependence on these quantities unless needed.*

Lemma B.8 *The following is always true for $\alpha \geq \alpha_{opaque}$:*

$$p_{ind} \geq p^*$$

Proof: Suppose otherwise. Then:

$$\Gamma_L(p^*) = \Sigma(p^*, p^*) < \Sigma(p_{ind}, p^*, w) \leq \Sigma(p_{ind}, p_{ind}, w) = \Gamma_L(p_{ind}, w)$$

Which is a contradiction since $(p^*, 0)$ maximizes $\Gamma_L(p, 0)$.

■

Lemma B.9 *There exists Δ such that if $Z_H^L - Z_L^L < \Delta$, then $\bar{p} > p_{ind}$ for all $\alpha > \alpha_{opaque}$.*

Proof: Consider $R_2(\alpha)$ and $R_1(\alpha)$, where these quantities are written as functions of α. I suppress dependence for notational convenience. From the definition of p_{ind}, it follows that $p_{ind} < \bar{p}$ if and only if:

$$\frac{Z_H^L}{Z_L^L} < \frac{1 - e^{-R_2(B^* - \bar{Q})}}{1 - e^{R_1(B^* - \bar{Q})}} \text{ where } \bar{Q} = \frac{e^{\bar{p}}}{1 + e^{\bar{p}}}$$

Since $\bar{p} = p_{max}$ for $\alpha \leq \alpha_{opaque}$ and is decreasing in α for $\alpha > \alpha_{opaque}$, it follows that $\frac{1 - e^{-R_2(B^* - \bar{Q})}}{1 - e^{R_1(B^* - \bar{Q})}}$ is increasing in α for $\alpha \geq \alpha_{opaque}$. L'Hospital's Rule yields:

$$lim_{\alpha \to \alpha_{opaque}^+} \frac{1 - e^{-R_2(B^* - \bar{Q})}}{1 - e^{R_1(B^* - \bar{Q})}} = \frac{R_2}{-R_1}$$

Thus, if $\frac{Z_H^L}{Z_L^L} < \frac{R_2(\alpha_{opaque})}{-R_1(\alpha_{opaque})}$, it follows that $p_{ind} < \bar{p}$ for all $\alpha \geq \alpha_{opaque}$.

■

B.4 Equilibrium: Proof of Theorem 6.2

I will characterize the equilibrium in all settings. As before, p^*, $\bar{p}$, and p_{ind} indicate the usual quantities. In an abuse of notation, I will suppress the dependence of these quantities on the wage, though the reader should understand that apart from p^*, the other two are functions of the wage offered by the low-tier firm. Let $\bar{p}_0$, and $p_{ind,0}$ denote the corresponding quantities when the wage is 0. Each is increasing in wage.

When making its decision, the high-tier firm will consider its belief about the hired worker and its observation of the offered wage. If $\Gamma_H(p, w) > \Pi_H(p_{max})$, it will operate on the secondary market. If $\Pi_H(p_{max}) > \Gamma_H(p,w)$, it will hire on the primary market at $t = 0$ and hire a high-signal worker. Based on this reasoning, I will only specify on-path outcomes and off-path beliefs when describing the equilibria. It is implicitly assumed that the high-tier firm will follow poaching rule τ^*.

Case #1: $p^*, \bar{p}_0 \geq \beta$

Under these conditions, the low-tier firm is incentivized to sort a little and hire high-signal workers. Pure strategy equilibrium will exist. Whether the pure strategy equilibrium involves F_H monitoring in the secondary market or not depends on the relationship between p^* and $\bar{p}$. Define $w_i(t) = \Gamma_H^{-1}(M_{high}(t), \Pi_H(p_{max}))$ to be the wage the low-tier firm needs to pay to make the high-tier firm indifferent between operating on the secondary market and hiring on primary market.

Suppose $\Pi_L(\bar{p}) > \Gamma_L(p^*)$:

This is trivially satisfied when $\bar{p} > p^*$. Let $\bar{t}_0$ be the time such that $M_{high}(\bar{t}_0) = \bar{p}_0$. Trivially, $w_i(\bar{t}_0) = 0$. Next, define $t_{eq} = argmax_{t \geq \bar{t}_0} \Pi_L(M_{high}(t), w_i(t))$. This will be the equilibrium hiring time for the low-tier firm.

The equilibrium strategies are as follows:

1. F_L hires a high-signal worker at time t_{eq} and offers wage $w_i(t_{eq})$.

2. F_H hires a high-signal worker at time $t = 0$. If it observes that F_L hires at any time $t < \bar{t}_0$, it also hires at $t = 0$. If it observes F_L hire in $t > \bar{t}_0$, it makes its decision based on the wage it observes. In other words, F_H will hire at $t = 0$ if and only if $\Pi_H(p_{max}) \geq \Gamma_H(M_{high}(t), w)$.

This equilibrium is clearly a PBE. On path, F_L will hire a high-signal worker at time t_{eq} and F_L will hire a high-signal worker at time $t = 0$. For moderately transparent environments, t_{eq} will be strictly less than 0, demonstrating that not only will there be a pure strategy unraveling equilibrium, but flexible wage-setting is not enough to stop unraveling from happening. An example where this could happen is when Z_H^L is much smaller than Z_H^H or if the intensity of worker preference for the high-tier firm (δ) is large. Then the maximum wage a low-tier firm could conceivably offer, Z_H^L, will have a marginal effect.

Suppose $\Gamma_L(p^*) > \Pi_L(\bar{p}_0)$:

It is necessary that $p^* > \bar{p}$. This corresponds to a scenario when the low-tier firm may be ok with screening more, even though it means getting poached. Intuitively, this occurs when the high-tier firm's payoff from primary market hiring is close to 0, and the differential between the low-tier firm's preferences over worker types is large.

Let t^* be the time such that $M_{high}(t^*) = p^*$. Define $\hat{t} = argmax_{t \geq \bar{t}_0} \Pi_L(M_{high}(t), w_i(t))$. The equilibrium payoff to the low-tier firm is:

$$\max\left\{\Gamma_L(p^*, 0), \Pi_H(M_{high}(\hat{t}), w_i(\hat{t}))\right\}$$

If the maximal value is $\Gamma_L(p^*, 0)$, then the equilibrium involves the low-tier firm hiring at t^*. By Lemma B.4, the equilibrium wage will be 0. The high-tier firm operates on the primary market with belief p^*. Otherwise, the low-tier firm hires at time $\hat{t}$ and offers a wage $w_i(\hat{t})$. The high-tier firm hires at the end of the primary market.

Case #2: $\bar{p} \geq \beta > p^*$ or $p^* > \beta \geq \bar{p}$

The corresponding pure strategy equilibrium found in the previous case is the unique pure strategy equilibrium here.

NOTE 3 *While I break indifference in the above cases by assuming the high-tier firm operates on the primary market, this is not necessary. Any mixing will result in the same equilibrium payoffs (increase the wage by ε and take the limit at ε goes to 0).*

Case #3: $p^*, \bar{p} < \beta$

An important feature of this case is that "*within* time deviations" must be considered. In the previous cases, when the low-tier firm was incentivized to hire high signal workers, the only way it could do better was to change the time it interviewed. Such a deviation is observable to the high-tier firm. Now, a low-tier firm incentivized to hire low-signal workers can deviate within the same hiring time. If the high-tier firm has a low belief about the worker, the low-tier firm may deviate to hire a high signal worker! As a result, there is no pure strategy equilibrium where F_H hires at the end of the primary market with probability 1. The reason for this is that in any such candidate equilibrium, F_L would need to hire a worker at time t that is high type with probability less than or equal to $\bar{p}$. However, if F_H is hiring at the end of the primary market, F_L has no incentive to hire such a worker. It can achieve a higher payoff by hiring the worker that is high type with probability $M_{high}(t) > \beta > \bar{p}$.

NOTE 4 *Since $p_{ind} \geq p^*$ by Lemma B.8, I can ignore p^* in the rest of the analysis.*

Using the functions constructed in Lemma B.10 on the following page, it is clear that F_H must follow $\chi_{prim}(w,t)$. Its beliefs both on and off-path are described by $\hat{p}(w,t)$. For the low-tier firm, define the set D of possible equilibrium hiring times:

$$D = \arg max_{t\in[-T,0]} \chi_{prim}(w^*,t)\Pi_L(\hat{p}(w^*,t),w^*) + (1-\chi_{prim}(w,t))\Gamma_L(\hat{p}(w^*,t),\hat{p}(w^*,t),w^*)$$

Therefore all equilibria can be described as follows:

1. F_H hires in the primary market with probability $\chi_{prim}(w,t)$, where w is the wage paid by the low-tier firm and t is the time at which the low-tier firm hired the worker. F_H has belief $\hat{p}(w,t)$ about the worker.

2. F_L chooses a distribution over D and, based on the realization, selects a high-signal worker with probability $\chi_{hi}(t)$ and a low-signal worker with probability $1-\chi_{hi}(t)$. The worker is paid $w^*(t)$.

■

Lemma B.10 *Fix an α and consider the setting where $\bar{p}, p_{ind} < \beta$. Suppose the low-tier firm is constrained to hiring at time t. The equilibrium in this constrained game is given by a function $\chi_{prim}(w,t)$ representing the probability the high-tier firm hires in the primary market. The low-tier firm hires a worker at wage $w^*(t)$, hiring a high-signal worker with probability $\chi_{hi}(t)$ and a low-signal worker with probability $1 - \chi_{hi}(t)$.*

Proof:

If the low-tier firm offers a wage w, it increases the poaching threshold to $\hat{B}^*$. The new indifference probability $p'_{ind} > p_{ind}$ satisfies $p'_{ind} - p_{ind} = B'^* - B^*$. Similarly, the indifference probability for the high-tier firm, $\bar{p}$, increases as well.

Define $\hat{p}(w,t) = \min\left\{\max\left\{p_{ind}(w), \bar{p}(w)\right\}, M_{high}(t)\right\}$. This represents the belief that must be induced in equilibrium in the constrained game.

Let $\hat{\chi}_{hi}(\hat{p}, w, t)$ denote the probability the low-tier firm hires a high-signal worker at time t given a wage of w. Let $\chi_{prim}(\hat{p}, w, t)$ be the probability the high-tier firm hires at the end of the primary markets after observing a wage w at time t. Since $\hat{p}$ depends on w and t, I suppress dependence of $\hat{\chi}_{hi}(\hat{p}, w, t)$ and $\chi_{prim}(\hat{p}, w, t)$ on $\hat{p}$.

The functions are defined as follows:

1. If $\hat{p}(w,t) > \bar{p}(w)$, $\chi_{prim}(\hat{p}, w, t) = 1$, and the high-tier firm believes the worker hired by the low-tier firm is high type with probability $\hat{p}(w,t)$.

2. The low-tier firm mixes so that $\hat{\chi}_{hi}(\hat{p}, w, t) M_{high}(t) + (1 - \hat{\chi}_{hi}(\hat{p}, w, t) = \hat{p}$.

The strategy χ_{prim} is clearly the equilibrium strategy for the high-tier firm. Now, the low-tier firm ultimately chooses $w^* \geq 0$ such that:

$$w^* = arg \max_{w} \chi_{prim}(w,t) \Pi_L(\hat{p}, w) + (1 - \chi_{prim}(w,t)) \Gamma_L(\hat{p}, w)$$

This w^* is unique. Defining $\chi_{hi}(t) = \hat{\chi}_{hi}(w^*, t)$ completes the proof.

∎

Relation to α

The three quantities $\{\bar{p}, p_{ind}, p^*\}$ are decreasing in α. By Lemmas B.5 and B.6, there is a threshold α_{high} such that for all $\alpha > \alpha_{high}$, $p_{ind} < \bar{p} < \beta$. As a result, in highly transparent environments, the equilibrium is in mixed strategies. Importantly, there is a mixed-strategy equilibrium with no unraveling. As $\alpha \to \infty$, $\bar{p} \to k$ where k satisfies $kZ_H^H = p_{max}Z_H^H + (1-p_{max})Z_L^H$. If $k \leq p_{min}$ then for α's sufficiently high, the equilibrium is in pure strategies. The high-tier firm monitors with probability 1, believing that the worker hired by F_L is of high type with probability p_{min}.

I provide an intuitive example to illustrate the equilibria as a function of α.

Example 3 *Consider an environment where $Z_H^L < \delta$. This means that the match quality to the low-tier firm from a high type worker is less than workers' preference for the high-tier firm. Therefore, wages will always be* 0*. Hence, $\alpha_{low} = \alpha_{opaque}$. Set $\alpha_{high} = \bar{p}^{-1}(\beta)$.*

For all $\alpha \in (\alpha_{opaque}, \alpha_{high}]$, the unique pure strategy equilibrium has the low-tier firm hiring at time $t_{eq} = M_{high}^{-1}(\max\{p^*, \bar{p}\})$. The market unravels.

If $Z_H^L - Z_L^L < \frac{R_2(\alpha_{opaque})}{-R_1(\alpha_{opaque})}$, then $\bar{p} > p^*$ for all $\alpha \geq \alpha_{opaque}$ by Lemma B.9. Therefore, for $\alpha \in (\alpha_{opaque}, \alpha_{high}]$, the unique pure strategy equilibrium consists of the low-tier firm hiring at time t_{eq} such that $M_{high}(t_{eq}) = \bar{p}$ and the high-tier firm hiring at the end of the primary market.

For $\alpha > \alpha_{high}$, the equilibrium outcome consists of the low-tier firm mixing between hiring high-signal and low-signal workers so that the worker is of high type with probability $\bar{p}$. The high-tier firm mixes between operating on the secondary market and hiring at the end of the primary market to ensure that the low-tier firm is indifferent between the type of worker it hires.

B.5 Proofs of Theorem 6.3 and 6.4

Proof of Theorem 6.3:

When $\alpha \leq \alpha_{opaque}$, the equilibrium is identical to the benchmark-setting, with equilibrium wages as 0. Therefore, I restrict attention to $\alpha > \alpha_{opaque}$.

The first condition in Theorem 6.3 rules out the existence of sufficiently high transparency levels where the equilibrium involves the high-tier firm operating in the secondary market with probability 1 (e.g. the low-tier firm is unable to hire a low-type worker with high enough probability to deter poaching). If the second condition in Theorem 6.3 holds, Lemma B.9 implies that $\bar{p} > p_{ind} \geq p^*$ for all α. Therefore, by Theorem 6.2, there are only pure strategy equilibria and equilibria in strict mixed-strategies.

The high-tier firm in all equilibria makes the same payoff as in the benchmark-setting: at every equilibrium, it is indifferent between operating on the secondary market and hiring at the end of the primary market. On the other hand, the low-tier firm makes strictly lower payoffs in any strict mixed-strategy equilibrium and pure-strategy unraveling equilibrium. In any pure strategy non-unraveling equilibrium for $\alpha > \alpha_{opaque}$, the wage is positive, and so the low-tier firm also earns a strictly lower payoff.

■

Proof of Theorem 6.4:

I demonstrate that coordination can reduce the low-tier firm's payoffs when the original equilibrium is a pure-strategy unraveling equilibrium.

Consider any primary and secondary market environment with the following properties:

1. $Z_H^L - Z_L^L < \dfrac{1+\sqrt{1+\frac{2r}{\alpha_{opaque}}}}{\sqrt{1+\frac{2r}{\alpha_{opaque}}}-1}$.

2. At some $\bar{\alpha}$, there exists a pure strategy unraveling equilibrium.

Since a pure strategy unraveling equilibrium exists when the transparency level is $\bar{\alpha}$, let $\bar{t}_0 < 0$ denote the time at which the low-tier firm hires. The hired worker is high type with probability $M_{high}(\bar{t}_0)$. In this pure-strategy unraveling equilibrium, the high-tier firm earns a payoff of $\Pi_H(p_{max})$. The low-tier firm earns a payoff of $\Pi_H(M_{high}(\bar{t}_0), w_i(\bar{t}_0))$.

If firms are constrained to hire and interview at $t = 0$, the new equilibrium must be in strict mixed strategies. Call these equilibria the *centralized equilibria*. An example of one is the following:

1. The low-tier firm mixes between hiring a low-signal and high-signal worker such that the net probability that the worker is of high-type is $\bar{p}_0 = M_{high}(\bar{t}_0)$. The wage is $w_i(\bar{t}_0)$. $*$See Theorem 6.2 for the definition of the function $w_i(\cdot)*$.

2. The high-tier firm F_H mixes between hiring on the primary market and operating on the secondary market. It mixes so that the low-tier firm is indifferent about which signal worker it hires. The high-tier firm believes that the low-tier firm's worker is high type with probability $\bar{p}_0$.

3. Off-path: if F_H observes a positive wage, it believes the low-tier firm has hired a high-signal worker, and so poaches as long as $\Gamma_H(p_{max}, w) \geq \Pi_H(p_{max})$.

The payoff to the low-tier firm in this centralized equilibrium is:

$$(1 - \chi_{prim})\Gamma_L(\bar{p}_0, w_i(\bar{t}_0)) + \chi_{prim}\Pi_L(\bar{p}_0, w_i(\bar{t}_0))$$

This is strictly lower than the equilibrium payoff in the unraveled market. This is not the only equilibrium. I will demonstrate that no matter the equilibrium one chooses, the low-tier firm will always receive a reduced payoff. The other centralized equilibria must involve the low-tier firm paying a positive wage w and hiring a worker with probability $\bar{p}(w)$ of being a high-type. The high-tier firm, though, must mix. Therefore, equilibrium payoffs for the low-tier firm are strictly lower than $\Pi_L(\bar{p}(w), w)$. Letting $t = w_i^{-1}(w)$, it follows that the payoffs are bounded by $\Pi_L(M_{high}(t), w_i(t))$. But this is bounded above by $\Pi_L(M_{high}(\bar{t}_0), w_i(\bar{t})$ as $\Pi_L(M_{high}(\bar{t}_0), w_i(\bar{t})$ is the payoff from the pure strategy unraveling equilibrium (see Theorem 6.2).

■

B.6 Robustness: Multiple Firms

Optimal Secondary Market Strategy:

Given N_H high-tier firms operating in the secondary market and $N_L > N_H$ low-tier firms operating in the primary market, I will characterize the optimal poaching rule for the high-tier firms. As before, I work in the belief space rather than the signal space.

Notation: Bold letters denote vectors. Let $V\left(\sigma, \mathbf{p_0}, k_h, k_l\right)$ be the value to a high-tier firm from strategy profile σ when there are k high-tier firms in the secondary market, l low-tier firms, and the current beliefs about the low-tier firms' workers are $\mathbf{p_0} \in [0,1]^l$.

I construct the optimal secondary market strategy recursively. Proof of optimality follows trivially by induction.

Case $N_H = 1$:

With a single high-tier firm, the optimal poaching rule is characterized by a continuation region $C \in [0,1]^{N_L}$. From Lemma B.2 and Dynkin (1969), the optimal poaching rule is the stopping time $\tau^{(1*)} = \inf_t \{t : \mathbf{p_t} \notin C\}$. Define $\sigma^{(1)}$ to be the strategy that corresponds to following $\tau^{(1*)}$.

Case $N_H = k$:

Let $q_t^{max} = \max \mathbf{p_t}$ be the N_L^{th} order statistic of the beliefs at time t. Define the vector $\mathbf{q_t^{min}} = \mathbf{p_t} \setminus \{q^{max}\}$ as the vector of beliefs at time t *excluding* the worker that all firms are most optimistic about. Define the stopping time $\tau^{(k*)}$:

$$\tau^{(k*)} = \inf_t \left\{ t : q_t^{max} \geq V\left(\sigma^{(k-1)}, \mathbf{q_t^{min}}, k-1, N_L - 1\right)\right\}$$

Consider the following strategy:

1. All firms poach at time $\tau^{(k*)}$. Since the firms approach the same worker, the worker randomly selects one to break the tie.

2. The remaining firms follow $\sigma^{(k-1)}$.

Label the strategy above as $\sigma^{(k)}$. For short-hand, $\sigma^{(k)} = \left\{\tau^{(k*)}, \sigma^{(k-1)}\right\}$.

■

Proof of Proposition 8.1:

Let $\mathbb{D}(p_i, \theta; \mathbf{p_{-i}})$ be the expected discounted probability of $F_{L,i}$ being poached when:

1. It employs a worker that is of type θ.

2. The initial belief about the worker being of high type is p_i.

3. All other low-tier firms hire a worker that is of high type with probability $\mathbf{p_{-i}}$.

$F_{L,i}$'s payoff is:

$$\Gamma_{L,i}(p_i; \mathbf{p_{-i}}) = \Pi_L(p_i) - \underbrace{\Big(p_i \mathbb{D}(p_i, \theta = H; \mathbf{p_{-i}}) Z_H^L + (1 - p_i) \mathbb{L}(p_i, \theta = L; \mathbf{p_{-i}}) Z_L^L\Big)}_{\mathbb{L}(p_i; \mathbf{p_{-i}})}$$

When $\frac{N_L}{N_H}$ large, the probability of an individual firm being poached is approximately $\frac{N_L}{N_H}$. Suppose all other firms hire a worker that is of high type with probability p_{max}. For any $\varepsilon \in (0,1)$, there exists J and K such that $\frac{N_L}{N_H} > J \textit{ and } N_L - N_H > K \implies$:

$$\frac{\mathbb{D}(p_i, \theta; \mathbf{p_{max}})}{p_i}\Big|_{p_i = p_{max}} < \varepsilon$$

$$\implies \frac{\partial \mathbb{L}(p_i; \mathbf{p_{max}})}{p_i}\Big|_{p_i = p_{max}} < \varepsilon (Z_H^L - Z_L^L)$$

Take $\varepsilon < p_0$ and the corresponding J and K. Under these conditions, $\Gamma_{L,i}(p_i; \mathbf{p_{max}})$ is maximized at $p_i = p_{max}$.

The expression for $\Gamma_{L,i}(p_{max}; \mathbf{p_{max}})$ implicitly assumes that the low-tier firms hire a worker that is of high type with probability p_{max}. As hiring a high-signal worker is not a guaranteed event, to ensure all the inequalities go through, it must be the case that the event that there are not enough high-signal workers happens with negligible probability. Consider a number $W(N_L, N_H) > 0$ large enough so that if $N > W(N_L, N_W)$, the probability of there being less than N_L high-signal workers is $\frac{p_0 - \varepsilon}{2}$. Such a $W(N_L, N_H)$ exists by the proof of Lemma 2.1.

■

C Primary and Secondary Market Microfoundation

C.1 Primary Market

If a firm chooses to hire in the primary market, it selects a time to conduct the interview and makes an offer based on the interview. The primary market must be such that the earlier a firm interviews, the lower its ability to sort between workers of high and low type. To capture this, I model interviewing as a probabilistic test on each worker that returns a *high* or *low* signal (denoted by lower-case h and l) depending on the true type of the worker. The associated conditional probabilities of a worker being of high type reflect the ability to sort at a given time t in the primary market.

Definition C.1 *Given* $x_h, x_l \in [0,1]$*, an* $(\mathbf{x_h}, \mathbf{x_l})$**-test** *is a signal applied to each worker that returns h (high) or l (low)*

$$P(h|\theta = H) = x_h$$

$$P(l|\theta = H) = x_l$$

Any (x_h, x_l)-test induces an ordered pair (p_h, p_l) where $p_h = P(\theta = H|h)$ and $p_l = P(\theta = H|l)$ are the conditional probabilities a worker is of high type given the results of the test. A *partial ordering* can be defined on the space of (x_h, x_l)-tests:

Definition C.2 *An* (x_h, x_l)*-test is* **more powerful** *than an* $(\hat{x}_h, \hat{x}_l)$*-test if and only if* $p_h \geq \hat{p}_h$ *and* $p_l \leq \hat{p}_l$*.*

Consider any mapping $Y : [-T, 0] \longrightarrow [0,1] \times [0,1]$ such that $Y(t)$ returns an (x_h, x_l)-test. Thus, Y associates with each time t a binary test that can be implemented for sorting between worker types. Since it is necessary to incorporate the feature that one can sort more effectively at later times, I impose the constraint that for such a Y to be admissible, it must be that $Y(t)$ is more powerful than $Y(t')$ for any t and t' such that $t \geq t'$. Call such a Y a testing-map.

It follows from Bayes' rule that any testing-map Y is equivalent to a unique mapping M. Likewise, any mapping M corresponds to a unique testing-map Y.

C.2 Secondary Market

For the definition of the secondary market to make sense, it must be the case that B_t is a Brownian motion with respect to all measures induced by $p_0 \in [0,1]$.

Lemma C.3 *There exists a probability space $(\Omega, \Sigma_\Omega, \mathbb{P}_0)$, where $\mathbb{P}_0 = p_0\mathbb{P}_H + (1 - p_0)P_L$, and a process B_t, such thatB_t is a Brownian motion with respect to all measures $\mathbb{P}_0$ induced by $p_0 \in [0,1]$.*

Proof: Consider the following two probability spaces:

1. A sufficiently rich space $(\mathbb{R}, \mathscr{B}, \lambda)$, where $\mathscr{B}$ is the Borel σ-algebra and λ is the lebesgue measure.
2. The space $(\{H,L\}, \Sigma, \hat{\mathbb{P}}_0)$, where $\hat{\mathbb{P}}(H) = p_0$, and Σ is the natural σ-algebra.

Let B_t be a brownian motion on $(\mathbb{R}, \mathscr{B}, \lambda)$. Let $\Omega = \{(\theta, a) : \theta \in \{H,L\} \text{ and } a \in \mathbb{R}\}$. Let Σ_Ω de the σ-algebra on Ω generated by Σ and $\mathscr{B}$. Finally, define the measure $\mathbb{P}_0$ to be the measure on (Ω, Σ) induced by $\hat{\mathbb{P}}_0$ and λ.

It follows that the measure $\mathbb{P}_0$ satisfies $\mathbb{P}_0(\theta = H) = p_0$ and that B_t is a brownian motion with respect to $(\Omega, \Sigma_\Omega, \mathbb{P}_0)$ for any $p_0 \in [0,1]$.

■

C.3 Signal Structure

It is crucial to point out that the idea of a binary test is just one interpretation of the mapping M. Randomization allows for the selection of a worker with probability of being a high type $p \in [M_{low}(t), M_{high}(t)]$. Thus, what is really assumed is that there is a cap on how well a firm can identify a high type worker at time t in the primary market. The mapping M is simply a reduced-form representation of this notion.

Example 4 *One could easily have an individual primary market signal for worker i, represented by a stochastic process $\pi^{prim,i}$ that evolves over time. To incorporate the* *idea that there is a cap on how well a firm can identify a high type worker at time t, the primary market signal for worker i would be:*

$$\pi_t^{prim,i} = \begin{cases} \overline{h}(t) & \pi_t^{(i)} \geq \overline{h}(t) \\ \pi_t^{(i)} & \pi_t^{(i)} \in (\underline{h}(t), \overline{h}(t)) \\ \underline{h}(t) & \pi_t^{(i)} \leq \underline{h}(t) \end{cases}$$

$$\textit{Where } \pi_t \textit{ satisfies } d\pi_t^{(i)} = \mu_\theta dt + \sigma_{prim} dB_t$$

$$\overline{h}(t) \textit{ is increasing, and } \underline{h}(t) \textit{ is decreasing}$$

Define the following functions $M_{high}(t)$ and $M_{low}(t)$:

$$M_{high}(t) = Prob\Big(\theta_i = H | \pi_{t'}^{prim,i} \geq \overline{h}(t') \textit{ for all } t' \leq t\Big)$$
$$M_{low}(t) = Prob\Big(\theta_i = H | \pi_{t'}^{prim,i} \leq \underline{h}(t') \textit{ for all } t' \leq t\Big)$$

The mapping $M(t) = (M_{high}(t), M_{low}(t))$ satisfies the properties in Section 2.2.

Now, within the secondary market, firms' ability to identify a high type worker is uncapped. The difference between the primary and secondary markets is that more information can be observed in the latter. Thus, my model applies to settings where there is a difference in the work done in each stage. Many labor markets have this feature since firms hire directly at the university level.

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